MathExcel Worksheet M: Final Exam Review

This review worksheet is NOT comprehensive, however the final exam will be. You should study from all materials available to you: book, notes, worksheets, old exams (including Exams 1–3), homework problems, and quizzes. The course calendar provides a list of topics and sections to study from.

1. Brighten your day by graphing the following parametric curve:

$$x(t) = 16\sin^3(t) \qquad y(t) = 13\cos(t) - 5\cos(2t) - 2\cos(3t) - \cos(4t) \qquad 0 \le t \le 2\pi$$

1 Extra practice with differential equations

2. Solve the following differential equations.

(a) $\dfrac{dy}{dx} = 6y^2x$

(b) $\dfrac{dy}{dx} = e^{-y}(2x-4)$

(c) $\dfrac{dy}{dx} = \dfrac{3x^2+4x-4}{2y-4}$

(d) $\dfrac{dy}{dx} = e^{y-x}\sec(y)$

(e) $\dfrac{dy}{dx} = \dfrac{xy^3}{\sqrt{1+x^2}}$

(f) $\dfrac{dy}{dx} = \dfrac{y}{x^2+x}$

3. Solve the initial value problem.

$$\frac{dy}{dx} = xe^y \qquad y(0) = 0$$

4. Solve the differential equation $\dfrac{dy}{dx} = x + y$ by making the substitution $u = x + y$.

2 Review of some less recent material

5. What does it mean for a sequence to converge? What does it mean for a series to converge?

6. Determine whether each series converges or diverges.

(a) $\displaystyle\sum_{n=1}^{\infty} e^{-n}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{3n^2+1}$ (c) $\displaystyle\sum_{n=1}^{\infty} n\sin\left(\frac{1}{n^2}\right)$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^n}$

7. Find the radius and interval of convergence for each power series.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{3^n}$ (b) $\displaystyle\sum_{n=1}^{\infty} nx^n$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 x^n}{n!}$

8. Give the Maclaurin series of each function, and state the interval of convergence.

(a) $f(x) = 4x^2 + x + 5$ (b) $f(x) = \dfrac{1}{x-1}$ (c) $f(x) = e^{2x}$

9. Evaluate each indefinite integral.

(a) $\displaystyle\int \frac{2}{x^2+2x}\,dx$ (c) $\displaystyle\int \frac{1}{\sqrt{1-x^2}}\,dx$ (e) $\displaystyle\int \sin^3(x)\,dx$

(b) $\displaystyle\int \sqrt{1+x^2}\,dx$ (d) $\displaystyle\int x\sin(x)\,dx$ (f) $\displaystyle\int x\cos(x^2)\,dx$

10. (a) Use the Trapezoid Rule, Midpoint Rule, and Simpson's Rule, each with 4 subintervals to approximate the area under the curve of $f(x) = x + \sin(x)$ on the interval $[0, 2\pi]$.

(b) Let T denote the Trapezoid Rule approximation from part (a), M the Midpoint Rule approximation, and S the Simpson's Rule approximation. Verify that

$$\frac{2M+T}{3} = S.$$

11. Find the volume of the solid obtained by revolving the ellipse $x^2 + \dfrac{y^2}{4} = 1$ about the line $x = 2$.

12. Consider the region $R = \{(x, y) \mid 0 \le y \le \sin(x), 2\pi \le x \le 3\pi\}$.

(a) Suppose that we revolve R about the line $x = 1$. Write an integral that gives the volume of the solid of revolution.

(b) Suppose that we revolve R about the line $y = -1$. Write an integral that gives the volume of the solid of revolution.