MA 114 MathExcel Worksheet I: Arc Length, Surface Area and Volumes

1. Calculate the arc length of the curve over the given interval.
   (a) $y = x^{3/2}$; $[1, 2]$
   (b) $y = \ln(\csc x)$; $\pi/4 \leq x \leq \pi/2$
   (c) $y = e^x$; $[0, 1]$

2. Set up the integral (do not compute) that gives the surface area for a revolution about the x-axis over the given interval.
   (a) $y = x^2$; $[0, 4]$
   (b) $y = e^{-x}$; $[0, 1]$
   (c) $y = \sin x$; $[0, \pi]$
   (d) $y = 4\cos(x^3)$; $[1, 2]$

3. Consider $f(x) = \frac{1}{4}x^2 - \frac{1}{2}\ln x$ over the interval $[1, e]$.
   (a) Calculate the arc length of $f(x)$.
   (b) Calculate the surface area of revolution about the x-axis of $f(x)$ over the interval.

4. Find the surface area of the torus obtained by rotating the circle $x^2 + (y - b)^2 = r^2$ about the x-axis.

5. The curves with equations $x^n + y^n = 1$ with $n = 4, 6, 8, \ldots$ are called *fat circles*.
   (a) Graph the curves with $n = 2, 4, 6$, and 8.
   (b) Set up the integral to find the length of the fat circle with $n = 2k$. Call this integral L_{2k}.
   (c) Based on your sketches from part (a), what is your guess for $\lim\limits_{k\to\infty} L_{2k}$?

6. Consider the parabola $y = x^2$ and only the points between $(2, 4)$ and $(3, 9)$. Sketch a picture and write an integral whose value is the length of this curve.

7. A hole of radius r is bored through the middle of a sphere of radius R. Set up two integral expressions for the volume of this solid using (i) the method of disks and (ii) the method of shells. Evaluate one of the integrals to find the volume.

8. A hole of radius r is bored through the central axis of a right circular cone with radius R and height H. Set up two integral expressions for the volume of this solid using (i) the method of disks and (ii) the method of shells. Evaluate one of the integrals to find the volume.

9. An auxiliary fuel tank for a helicopter is shaped like the surface generated by revolving the curve $y = 1 - \frac{x^2}{4}$, $-2 \leq x \leq 2$, about the x-axis (dimensions are in feet). Set up the integral that computes how many cubic feet of fuel the tank will hold.

10. Find the volume of the solid obtained by rotating the region bounded by $y = x^3$, $y = 0$, and $x = 1$ about the line $y = -3$. Leave your answer in exact form.

11. Let R be the region bounded by $y = e^{-x^2}, y = 0, x = 0$, and $x = 1$.
    (a) Using the method of cylindrical shells write down the integral needed to find the volume generated by rotating R about the y-axis.
    (b) Evaluate this intergal