MA 114 MathExcel Supplement - Worksheet H: Average Value and Volume

1. (a) State the Mean Value Theorem for Integrals

   (b) If f is continuous and $\int_1^3 f(x)\ dx = 8$, show that f takes on the value 4 at least once on the interval $[1, 3]$.

2. If a free falling body starts from rest, then its displacement at time t is given by $s = \frac{1}{2}gt^2$. Show that the average velocity on the interval $[0, T]$ is $v_{ave} = \frac{1}{2}v_T$, where v_T is the velocity of the object at time T.

3. Conceptual Understanding:

   a.) If a solid has a cross-sectional area given by the function $A(x)$, what integral should be evaluated to find the volume of the solid?

   b.) Suppose we wish to calculate the volume of the solid formed by rotating the graph of $y = f(x)$ between $x = a$ and $x = b$ around the x-axis. Explain why the following formula calculates this volume:

   $$V = \pi \int_a^b [f(x)]^2\ dx.$$

4. Find the volume of the solid whose base is the ellipse $9x^2 + 4y^2 = 36$ with cross sections of squares perpendicular to the x-axis.

5. Let V be the volume of a right circular cone of height 10 whose base is a circle of radius 4.

   a.) Use similar triangles to find a formula for the area of a horizontal cross section at a height y.

   b.) Use your answer to part (a) to calculate V.

6. Find the volume of the solid whose base is the region enclosed by the parabola $y = 1 - x^2$ and the x-axis with cross sections of equilateral triangles perpendicular to the x-axis.

7. Compute the volume of the solid whose base is the region between the inverted parabola $y = 4 - x^2$ and the x-axis, and whose cross sections perpendicular to the y-axis are semicircles.

8. Find the volume of revolution about the x-axis for $f(x) = \sqrt{\cos(x)\sin(x)}$ between $[0, \frac{\pi}{2}]$. (Hint: You will probably need to use u-substitution.)

9. Consider the region enclosed by $y = e^{-x}$, the x-axis, and the lines $x = 1$ and $x = 3$. What is the volume of the solid obtained by rotating this region around the line $y = -2$?

10. Find the volume of the solid obtained by rotating the region enclosed by $x = 4 - y$ and $x = 16 - y^2$ about:

    a.) the y-axis.

    b.) the line $x = -3$.

11. The base of a certain solid is the triangle with vertices at $(10, 5)$, $(5, 5)$, and the origin. Cross-sections perpendicular to the y-axis are squares. Find the volume of the solid.

12. Calculate the volume of the following solid. The base is a circle of radius r centered at the origin. The cross sections perpendicular to the x-axis are squares.

13. Set up an integral expression for the volume of a cap of a sphere with radius R and height H using the method of disks. Evaluate the integral to find the volume.