MA 114 MathExcel Worksheet G*: Exam 02 Review

1. It is easy to see that $\lim_{n\to\infty} \sin\left(\frac{1}{n}\right) = 0$. What does this say about the series $\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)$? Does the series converge or diverge? How can you tell?

2. Determine whether each sequence converges, and if so, to what value.

   (a) $\left\{\frac{1}{n}\right\}_{n=1}^{\infty}$ (b) $\left\{2^{n+1} - 2^n\right\}_{n=2}^{\infty}$ (c) $\left\{\frac{1}{n} - \frac{1}{n+1}\right\}_{n=1}^{\infty}$ (d) $\left\{1 + \frac{(-1)^n}{2^n}\right\}_{n=5}^{\infty}$

3. Determine whether each series converges, and if so, to what value.

   (a) $\sum_{n=0}^{\infty} \frac{2^n - 5^n}{8^n}$ (b) $\sum_{k=1}^{\infty} \left(\frac{1}{\sqrt{k}} - \frac{1}{\sqrt{k+1}}\right)$ (c) $\sum_{m=1}^{\infty} (\ln(m) - \ln(m+1))$

4. Show that the power series $f(x), g(x)$, and $h(x)$ below have the same radius of convergence. Then show that $f(x)$ diverges at both endpoints, $g(x)$ converges at one endpoint but diverges at the other, and $h(x)$ converges at both.

   $$f(x) = \sum_{n=1}^{\infty} \frac{x^n}{3^n} \qquad g(x) = \sum_{n=1}^{\infty} \frac{x^n}{n\, 3^n} \qquad h(x) = \sum_{n=1}^{\infty} \frac{x^n}{n^2\, 3^n}$$

5. Determine if each of the following series converges or diverges. For series that have some negative terms, specify whether the convergence is absolute or conditional. Bes sure to justify your answer.

   (a) $\sum_{j=0}^{\infty} \frac{2^{2j}}{j!}$

   (b) $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n + \ln(n)}}$

   (c) $\sum_{n=2}^{\infty} \frac{n}{\sqrt{n^5 + 5}}$

   (d) $\sum_{n=1}^{\infty} \frac{n^2}{(n^3+1)^{1.01}}$

   (e) $\sum_{n=1}^{\infty} \left(\frac{3}{4n}\right)^n$

   (f) $\sum_{n=1}^{\infty} \frac{\pi^{7n}}{e^{8n}}$

   (g) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n} + \sqrt{n+1}}$

   (h) $\sum_{k=1}^{\infty} \frac{1}{k + \sqrt{k}}$

   (i) $\sum_{n=2}^{\infty} \frac{1}{n^{\ln(n)}}$

   (j) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\ln(n)}$

6. Find the interval of convergence of each of the following power series. Remember to check endpoints!

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{n^6}{n^8+1}(x-3)^n$ (b) $\displaystyle\sum_{n=0}^{\infty}(nx)^n$ (c) $\displaystyle\sum_{n=2}^{\infty} \frac{(x+4)^n}{(n\ln(n))^2}$

7. Recall that $\sinh(x) = \frac{1}{2}(e^x - e^{-x})$ and $\cosh(x) = \frac{1}{2}(e^x + e^{-x})$.

   (a) Find the Maclaurin series representations of $\sinh(x)$ and $\cosh(x)$.

   (b) What are the intervals of convergence of the series you found in part (a)?

   (c) Use the Maclaurin series to verify that $\frac{d}{dx}(\sinh(x)) = \cosh(x)$ and $\frac{d}{dx}(\cosh(x)) = \sinh(x)$.

8. Find a power series representation for each function and determine its radius of convergence.

   (a) $f(x) = \dfrac{5}{1-4x^2}$

   (b) $f(x) = \dfrac{x^2}{x^4+16}$

   (c) $f(x) = \dfrac{3}{2+2x}$

   (d) $f(x) = e^{-x^2}$

9. Using the formula

$$\ln(1+x) = \int_0^x \frac{1}{1+t}\,dt$$

find a power series for $\ln(1+x)$ and state its radius of convergence.

10. Use the Maclaurin series for $\cos(x)$ to compute

$$\lim_{x\to 0} \frac{1-\cos(x)}{x^2}.$$