MA 114 MathExcel Supplement - Worksheet F: Convergence/Divergence Tests & Power Series

1. Concept Check: Write out all hypothesis and conclusions for the following tests.
   - Integral Test
   - Comparison Test
   - Limit Comparison Test
   - Alternating Series Test (Conditional vs Absolute Convergence)
   - Ratio Test
   - Root Test

2. Determine the values of $\alpha \in \mathbb{R}$ for which

$$\sum_{n=1}^{\infty} \left(\frac{\alpha n}{n+1} \right)^n$$

   converges.

3. In each of the following cases, discuss the convergence/divergence of the series $\sum_{n=1}^{\infty} a_n$ where a_n equals:

   (a) $\dfrac{n!}{e^{n^2}}$

   (b) $\dfrac{n^2 2^n}{(2n+1)!}$

   (c) $\left(1 - \dfrac{1}{n}\right)^{n^2}$

   (d) $\dfrac{n^2}{3^n}\left(1 + \dfrac{1}{n}\right)^{n^2}$

   (e) $\sin\left(\dfrac{(-1)^n}{n^p}\right)$, $p > 0$

   (f) $(-1)^n \dfrac{(\ln n)^3}{n}$

   (g) $(-1)^n \left(n^{\frac{1}{n}} - 1\right)^n$

   (h) $\dfrac{2^n + n^2 - \ln n}{n!}$

   (i) $\dfrac{\cos(\pi n) \ln n}{n}$

   (j) $\left(1 + \dfrac{2}{n}\right)^{n^2 - \sqrt{n}}$

   (k) $\dfrac{n^2 \left(2\pi + (-1)^n\right)^n}{10^n}$

4. What is the difference between a Series and a Power Series? What is the "Radius of Convergence" and how do you find it?

5. Determine the Radius of Convergence for the following series.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-2)^n n}{5^n}(x+3)^n$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2^n}{n}(4x-8)^n$

   (c) $\displaystyle\sum_{n=1}^{\infty} n!(2x+1)^n$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^n}$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n}}{(-3)^n}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{(2x)^{2n}}{(-5)^n}$

   (g) $\displaystyle\sum_{n=1}^{\infty} x^n$

   (h) $\displaystyle\sum_{n=1}^{\infty} (x-a)^n$

   (i) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n}}{(-3)^n}$

6. Use your knowledge of Geometric series to write the following as a series, then find the radius of convergence.

   (a) $\frac{4}{1-x}$

   (b) $\frac{1}{1-x^2}$

   (c) $\frac{1}{2x-x^2}$

7. Consider the power series $\displaystyle\sum_{n=0}^{\infty} \frac{x^n}{n!}$.

   (a) Find the radius and interval of convergence of this power series.

   (b) Express the derivative of this power series in summation notation. (You may find it helpful to write out the first few terms.)

   (c) Express the antiderivative of this power series (with $C = 1$) in summation notation. (Again, you may find it helpful to write out the first few terms.)

   (d) What do you notice about the original series and your answers to parts (b) and (c)?

   (e) Can you think of a function we know well that also has this property?