# MA 114 MathExcel Worksheet E: Series Tests and Absolute/Conditional Convergence

1. State or define each of the following (with appropriate hypotheses):

   (a) Test for Divergence
   (b) Integral Test
   (c) Geometric Series
   (d) Comparison Test
   (e) Limit Comparison Test
   (f) Absolute Convergence
   (g) Conditional Convergence
   (h) Alternating Series Test

2. For large $n$, rank each set of functions in increasing order. (For example, for large enough $n$, we can say $n^2 < n^3$). Explain how your results might be helpful in determining whether or not a series converges.

   (a) $n!$, $e^n$, and $\cos(n\pi)$
   (b) $(n-1)^3$, $\ln(n)$, and $10^n$
   (c) $n$, $n^{10}$, and $n^{0.1}$

3. Ron hopes to investigate the convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{e^{-n}}{n}$ by comparing it with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$. Hermione suggests that this might be a bad idea. Why is Hermione right (this time)?

4. Consider the sum $\dfrac{1}{2} - \dfrac{1}{3} + \dfrac{1}{2^2} - \dfrac{1}{3^2} + \dfrac{1}{2^3} - \dfrac{1}{3^3} + \cdots$

   (a) Can you apply the Alternating Series Test to this series? Why or why not?
   (b) Show that this series converges.

5. Consider the series
$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+1}$$

   (a) For $b_n = \frac{n^2}{n^3+1}$, show that $b_{n+1} \leq b_n$ for all $n$ and $\lim_{n\to\infty} b_n = 0$.
   (b) Does the series congverge or diverge?

6. For the following $a_n$, determine first if $\{a_n\}$ converges, then determine if $\sum_{n=1}^{\infty} a_n$ converges.

   (a) $a_n = \dfrac{2}{n}$.
   (b) $a_n = \dfrac{5n^4+17}{n^{13}-6}$.
   (c) $a_n = 18$.

7. Determine whether the following series converge absolutely, conditionally or not at all.

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n}$

(b) $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^{1/3}}$

(c) $\sum_{n=2}^{\infty} \frac{\cos(\pi n)}{\ln(n)}$

(d) $\frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} + \frac{1}{4} - \frac{1}{4} + \cdots$

(e) $2 - 2 + 3 - 3 + 4 - 4 + \cdots$

8. Using the method of your choice, determine whether the following series converge or diverge.

(a) $\sum_{n=0}^{\infty} \pi^{-n}$

(b) $\sum_{n=1}^{\infty} \frac{1}{3n^4 + 12n}$

(c) $\sum_{n=1}^{\infty} \frac{3^n + (-2)^n}{7^n}$

(d) $\sum_{n=1}^{\infty} (-1)^n n^2 e^{-n^3/3}$

(e) $\sum_{n=1}^{\infty} \frac{n^2}{n^4 - 1}$

(f) $\sum_{n=1}^{\infty} \frac{e^n + n}{e^{2n} - n^2}$

(g) $\sum_{n=1}^{\infty} \frac{1}{n^n}$

(h) $\sum_{n=1}^{\infty} \frac{10^n}{2^{n^2}}$

(i) $\sum_{n=0}^{\infty} \frac{(-1)^n}{5n + 1}$

(j) $\sum_{n=1}^{\infty} \frac{1}{2^n - 1}$