MathExcel Worksheet D: Introduction to Infinite Series

1. Comprehension Check:

   (a) What does it mean to say an infinite series converges?

   (b) State the Test for Divergence. Can you ever use this to show that a series converges?

   (c) What is a geometric series? When does that series converge? To what value does it converge?

   (d) State the Integral Test.

2. For each of the following series, find the first four terms and the first four partial sums.

   (a) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

   (b) $\frac{4}{5} - \frac{6}{7} + \frac{8}{9} - \frac{10}{11} + \ldots$

3. Use the divergence test to prove that each of the following series diverge.

   (a) $\sum_{n=1}^{\infty} \frac{n}{10n+12}$

   (b) $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^2+1}}$

   (c) $\sum_{n=1}^{\infty} \cos\left(\frac{1}{n}\right)$

   (d) $\sum_{n=1}^{\infty} (\sqrt{4n^2+1} - n)$

4. Use the formula for the sum of a geometric series to find the sum or state that the series diverges.

   (a) $\sum_{n=1}^{\infty} (e)^{-n}$

   (b) $\sum_{n=0}^{\infty} \frac{8+2^n}{5^n}$

   (c) $\frac{7}{8} - \frac{49}{64} + \frac{343}{512} - \frac{2401}{4096} + \ldots$

   (d) $\frac{25}{9} + \frac{5}{3} + 1 + \frac{3}{5} + \frac{9}{25} + \frac{27}{125} + \ldots$

5. Consider the following series: $\sum_{n=1}^{\infty} \frac{1}{n(n+4)}$.

   (a) Use partial fraction decomposition to expand $\frac{1}{n(n+4)}$.

   (b) Write out a few partial sums and find a closed form expression for $S_N = \sum_{n=1}^{N} \frac{1}{n(n+4)}$.

   (c) Find $\sum_{n=1}^{\infty} \frac{1}{n(n+4)}$ by taking the limit of your expression in part (b).

6. Show that if a is a positive integer, then $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+a)} = \frac{1}{a}\left(1 + \frac{1}{2} + \ldots + \frac{1}{a}\right)$.

7. Use the Integral Test to show that the harmonic series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$ diverges.

8. Let $b_n = \dfrac{\sqrt[n]{n!}}{n}$.

   (a) Show that $\ln b_n = \displaystyle\frac{1}{n}\sum_{k=1}^{n} \ln\frac{k}{n}$.

   (b) Show that the sequence $\{\ln b_n\}_{n=1}^{\infty}$ converges to $\displaystyle\int_0^1 \ln(x)\,dx$ (which is equal to -1).

   (c) What is $\displaystyle\lim_{n\to\infty} b_n$?