MathExcel Worksheet C*: Exam I Review

Sequences:

1. Write the first four terms of the sequences with the following general terms:

   (a) $\{a_n\}_{n=1}^{\infty}$ where $a_n = \dfrac{3}{n}$.

   (b) $\{b_n\}_{n=1}^{\infty}$ where $b_n = (-1)^n$

   (c) $\{c_n\}_{n=1}^{\infty}$ where $c_n = 2^{-n} + 2$.

   (d) $\{d_k\}_{k=1}^{\infty}$ where $d_k = \dfrac{(-1)^k}{k^2}$.

2. Find a formula for the nth term of each sequence.

   (a) $\left\{\dfrac{1}{1}, -\dfrac{1}{8}, \dfrac{1}{27}, -\dfrac{1}{64}, \dots\right\}$

   (b) $\left\{1, \dfrac{1}{2}, \dfrac{1}{4}, \dfrac{1}{8}, \dfrac{1}{16}, \dots\right\}$

   (c) $\{1, 0, 1, 0, 1, 0, \dots\}$

   (d) $\left\{-\dfrac{1}{2}, \dfrac{2}{3}, -\dfrac{3}{4}, \dfrac{4}{5}, -\dfrac{5}{6}, \dots,\right\}$

3. Suppose that a sequence $\{a_n\}$ is bounded above and below. Does it converge? If not, find a counterexample.

4. The limit laws for sequences are the same as the limit laws for functions. Suppose you have sequences $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ with $\lim_{n\to\infty} a_n = 15$, $\lim_{n\to\infty} b_n = 0$ and $\lim_{n\to\infty} c_n = 1$. Use the limit laws of sequences to answer the following questions.

   (a) Does the sequence $\left\{\dfrac{a_n \cdot c_n}{b_n + 1}\right\}_{n=1}^{\infty}$ converge? If so, what is its limit?

   (b) Does the sequence $\left\{\dfrac{a_n + 3 \cdot c_n}{2 \cdot b_n + 2}\right\}_{n=1}^{\infty}$ converge? If so, what is its limit?

Integration Strategies:

5. For each of the following integrals, decide which method you should use to integrate. Verify that your strategy will work by working out the problem until you're sure you'll get to the right answer. If you need more practice on this kind of problem, do the whole thing. *Tl;dr: Make sure you know how to do these problems, but don't waste time doing computations if you're sure you know what you're doing.*

   (a) $\displaystyle\int x^2 \sin 2x\, dx$

   (b) $\displaystyle\int xe^{2x}\, dx$

   (c) $\displaystyle\int \frac{dx}{x^2 + 2x + 10}$

   (d) $\displaystyle\int \frac{x+3}{(x-6)(x-3)}\, dx$

   (e) $\displaystyle\int \frac{3x+6}{x^2 - 10x + 24}\, dx$

   (f) $\displaystyle\int \frac{3x^2 + 9x + 8}{x^2(x+2)^2}\, dx$

   (g) $\displaystyle\int \sin^5 x \cos x\, dx$

   (h) $\displaystyle\int \sin^2 x\, dx$

   (i) $\displaystyle\int \frac{dx}{x\sqrt{x^2+9}}$

   (j) $\displaystyle\int \sqrt{16 + 4x^2}\, dx$

   (k) $\displaystyle\int x^3\sqrt{9 - x^2}\, dx$

   (l) $\displaystyle\int_1^2 \frac{dx}{x \ln x}$

   (m) $\displaystyle\int_1^{\infty} xe^{-2x}\, dx$

   (n) $\displaystyle\int x \tan(2x)\sec(2x) dx$

   (o) $\displaystyle\int \tan^3(x)\sec^2(x) dx$

(p) $\displaystyle\int_1^{\sqrt{3}} \arctan(1/x)dx$ (Hint: integrate by parts $dv = dx$) with $u = \arctan(1/x)$ and

6. Find the partial fraction decomposition of the following rational functions. Do **NOT** evaluate any integrals (unless you really want to).

   (a) $\dfrac{x^2+4x+12}{(x+2)(x^2+4)}$

   (b) $\dfrac{x^4-4x+8}{x^3+2x^2+4x+8} = \dfrac{x^4-4x+8}{(x+2)(x^2+4)}$

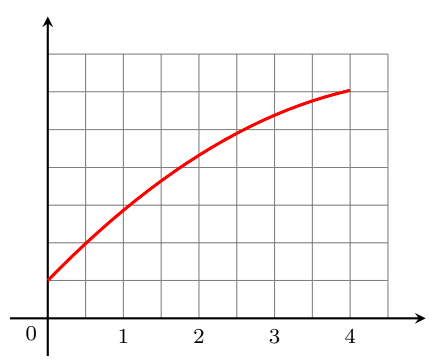
Numerical Integration:

7. Let $f = \frac{1}{12}x^6+2x^4-7x+2$ on $[-1,5]$. Given that the error in the Simpson's rule approximation of $\int_a^b f(x)\,dx$ satisfies
$$|E_S| \leq \frac{K(b-a)^5}{180n^4}$$
where $|f^{(4)}(x)| \leq K$ on $[a,b]$, determine how many subintervals are required to approximate $\displaystyle\int_{-1}^{5} f(x)\,dx$ with an error of less than .0002.

8. A table of values for a continuous function f is shown below. If four equal subintervals of $[0,2]$ are used, what is the trapezoid rule approximation for $\displaystyle\int_0^2 f(x)\,dx$?

x	0.0	0.5	1.0	1.5	2.0
f(x)	2	8	6	12	10

9. Let $I = \int_0^4 f(x)\,dx$, where f is the function whose graph is shown below. For any value of n, list the numbers L_n, R_n, M_n, and T_n in increasing order.



Improper Integrals:

10. For each of the following integrals, decide which is improper. For the improper integrals, set up **but do not evaluate** the corresponding limit problem.

(a) $\displaystyle\int_{-\infty}^{3} x^2 \, dx$

(b) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2} \, dx$

(c) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \tan\theta \, d\theta$

(d) $\displaystyle\int_{0}^{10000} \ln(x^2+1) \, dx$

11. For the integrals below, determine if the integral is convergent or divergent. Evaluate the convergent integrals.

(a) $\displaystyle\int_{1}^{\infty} \frac{1}{x^{3/2}} \, dx$

(b) $\displaystyle\int_{-\infty}^{\infty} xe^{-x^2} \, dx$

(c) $\displaystyle\int_{4}^{\infty} \frac{1}{x} \, dx$

(d) $\displaystyle\int_{-1}^{0} \frac{e^{1/x}}{x^3} \, dx$

(e) $\displaystyle\int_{0}^{2} \frac{x}{x-1} \, dx$