MathExcel Worksheet B:
Partial Fractions and Numerical Integration

1. Evaluate the following indefinite integrals.

   (a) $\displaystyle\int \frac{e^x}{e^{2x}+3e^x+2}\,dx$

   (b) $\displaystyle\int \frac{x^3-4x+1}{x^2-3x+2}\,dx$

   (c) $\displaystyle\int \frac{y}{(y+4)(2y-1)}\,dy$

2. Evaluate

$$\int \frac{1}{x^2+k}\,dx$$

   where k is a constant. If you aren't sure where to start, try evaluating the integral for a specific value of k.

3. Consider the integral: $\displaystyle\int_{-1}^{1} \ln(1+x^2)\,dx$

   (a) Use the trapezoid rule with $n=4$ to approximate the integral.

   (b) Use Simpson's rule with $n=6$ to approximate the integral.

4. Consider the integral $\displaystyle\int_0^2 \sqrt{x}\,dx$.

   (a) Evaluate the integral exactly.

   (b) What is the largest error you would expect from an approximation using the trapezoid rule? Using Simpson's rule?

   (c) Use the trapezoid rule with $n=6$ to approximate the integral. Is this an overestimate or underestimate? Calculate the error.

   (d) Use Simpson's rule with $n=6$ to approximate the integral. Calculate the error.

   (e) How many subintervals (i.e. what value of n) are needed to guarantee a Midpoint approximation within 0.0001 of the exact value?

5. Numerical integration allows us to approximate irrational numbers. Consider the integral $\displaystyle\int_0^1 \frac{4}{1+x^2}\,dx$.

   (a) What is the exact value of the integral?

   (b) Given that $|f^{(4)}(x)| \le 96$ for $x \in [0,1]$, use Simpson's rule to approximate the value within 0.0001.

6. For each of the following integrals, decide which is improper. For the improper integrals, set up BUT DO NOT EVALUATE the corresponding limit problem.

   (a) $\displaystyle\int_{-\infty}^{3} x^2\,dx$

   (b) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx$

   (c) $\displaystyle\int_0^1 \frac{x}{x^2+3}\,dx$

   (d) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{2\pi}{3}} \tan\theta\,d\theta$

   (e) $\displaystyle\int_{-2}^{2} \frac{t}{\sqrt{9-t^2}}\,dt$

   (f) $\displaystyle\int_0^{10000} \ln(x^2+1)\,dx$

(g) $\displaystyle\int_{-\infty}^{\infty} \frac{1}{s^2+2s-15}\, ds$

(h) $\displaystyle\int_{-1}^{1} \frac{\sin y}{\sqrt{y^2-y}}\, dy$