MA 114 Worksheet #08*: Sequences

1. (a) Give the precise definition of a **sequence**.
   (b) What does it mean to say that $\lim_{x\to a} f(x) = L$ when $a = \infty$? Does this differ from $\lim_{n\to\infty} f(n) = L$? Why or why not?
   (c) What does it means for a sequence to converge? Explain your idea, not just the definition in the book.
   (d) Sequences can diverge in different ways. Describe two distinct ways that a sequence can diverge.
   (e) Give two examples of sequences which converge to 0 and two examples of sequences which converges to a given number $L \neq 0$.

2. Match each sequence with its general term:

$\{a_1, a_2, a_3, a_4, ...\}$	General Term
(a) $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, ...\right\}$	(i) $\cos(\pi n)$
(b) $\{-1, 1, -1, 1, ...\}$	(ii) $\frac{n!}{2^n}$
(c) $\{1, -1, 1, -1, ...\}$	(iii) $(-1)^{n+1}$
(d) $\left\{\frac{1}{2}, \frac{2}{4}, \frac{6}{8}, \frac{24}{16}, ...\right\}$	(iv) $\frac{n}{n+1}$

3. Let $a_n = \dfrac{1}{2n-1}$ for $n = 1, 2, 3, \cdots$. Write out the first three terms of the following sequences.

   (a) $b_n = a_{n+1}$
   (b) $c_n = a_{n+3}$
   (c) $d_n = a_n^2$
   (d) $e_n = 2a_n - a_{n+1}$

4. Suppose that $\lim_{n\to\infty} a_n = 4$ and $\lim_{n\to\infty} b_n = 7$. Determine the following:

   (a) $\lim_{n\to\infty} (a_n + b_n)$
   (b) $\lim_{n\to\infty} a_n^3$
   (c) $\lim_{n\to\infty} \cos(\pi b_n)$
   (d) $\lim_{n\to\infty} (a_n^2 - 2a_n b_n)$

5. Suppose you know that $\{a_n\}$ is a decreasing sequence with $5 \leq a_n \leq 8$ for all a_n. Why must this sequence have a limit? What can you say about the value of the limit?