MA 114 Worksheet #07: Sequences

1. (a) Give the precise definition of a **sequence**.
   (b) What does it mean to say that $\lim_{x \to a} f(x) = L$ when $a = \infty$? Does this differ from $\lim_{n \to \infty} f(n) = L$? Why or why not?
   (c) What does it means for a sequence to converge? Explain your idea, not just the definition in the book.
   (d) Sequences can diverge in different ways. Describe two distinct ways that a sequence can diverge.
   (e) Give two examples of sequences which converge to 0 and two examples of sequences which converges to a given number $L \neq 0$.

2. Write the first four terms of the sequences with the following general terms:
   (a) $\frac{n!}{2^n}$
   (b) $\frac{n}{n+1}$
   (c) $(-1)^{n+1}$
   (d) $\{a_n\}_{n=1}^{\infty}$ where $a_n = \frac{3}{n}$.
   (e) $\{a_n\}_{n=1}^{\infty}$ where $a_n = 2^{-n} + 2$.
   (f) $\{b_k\}_{k=1}^{\infty}$ where $b_k = \frac{(-1)^k)}{k^2}$.

3. Find a formula for the nth term of each sequence.
   (a) $\left\{ \frac{1}{1}, -\frac{1}{8}, \frac{1}{27}, -\frac{1}{64}, \ldots \right\}$
   (b) $\left\{ 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots \right\}$
   (c) $\{1, 0, 1, 0, 1, 0, \ldots\}$
   (d) $\left\{ -\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, -\frac{5}{6}, \ldots, \right\}$

4. Suppose that a sequence $\{a_n\}$ is bounded above and below. Does it converge? If not, find a counterexample.

5. The limit laws for sequences are the same as the limit laws for functions. Suppose you have sequences $\{a_n\}$, $\{b_n\}$ and $\{c_n\}$ with $\lim_{n\to\infty} a_n = 15$, $\lim_{n\to\infty} b_n = 0$ and $\lim_{n\to\infty} c_n = 1$. Use the limit laws of sequences to answer the following questions.
   (a) Does the sequence $\left\{ \frac{a_n \cdot c_n}{b_n + 1} \right\}_{n=1}^{\infty}$ converge? If so, what is its limit?
   (b) Does the sequence $\left\{ \frac{a_n + 3 \cdot c_n}{2 \cdot b_n + 2} \right\}_{n=1}^{\infty}$ converge? If so, what is its limit?

Math Excel Worksheet #7: Introduction to Sequences

1. Match each sequence with its general term:

$\{a_1, a_2, a_3, a_4, ...\}$	General Term
(a) $\left\{\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, ...\right\}$	(i) $\cos(\pi n)$
(b) $\{-1, 1, -1, 1, ...\}$	(ii) $\frac{n!}{2^n}$
(c) $\{1, -1, 1, -1, ...\}$	(iii) $(-1)^{n+1}$
(d) $\left\{\frac{1}{2}, \frac{2}{4}, \frac{6}{8}, \frac{24}{16}, ...\right\}$	(iv) $\frac{n}{n+1}$

2. Let $a_n = \dfrac{1}{2n-1}$ for $n = 1, 2, 3, \cdots$. Write out the first three terms of the following sequences.

   (a) $b_n = a_{n+1}$

   (b) $c_n = a_{n+3}$

   (c) $d_n = a_n^2$

   (d) $e_n = 2a_n - a_{n+1}$

3. Suppose that $\lim_{n\to\infty} a_n = 4$ and $\lim_{n\to\infty} b_n = 7$. Determine the following:

   (a) $\lim_{n\to\infty} (a_n + b_n)$

   (b) $\lim_{n\to\infty} a_n^3$

   (c) $\lim_{n\to\infty} \cos(\pi b_n)$

   (d) $\lim_{n\to\infty} (a_n^2 - 2a_n b_n)$

4. Suppose you know that $\{a_n\}$ is a decreasing sequence with $5 \le a_n \le 8$ for all a_n. Why must this sequence have a limit? What can you say about the value of the limit?