MA 114 Worksheet #05: Numerical Integration

1. (a) Write down the Midpoint rule and illustrate how it works with a sketch.

   (b) Write down the Trapezoid rule, illustrate how it works with a sketch, and write down the error bound associated with it.

   (c) How large should n be in the Midpoint rule so that you can approximate

$$\int_0^1 \sin(x)\,dx$$

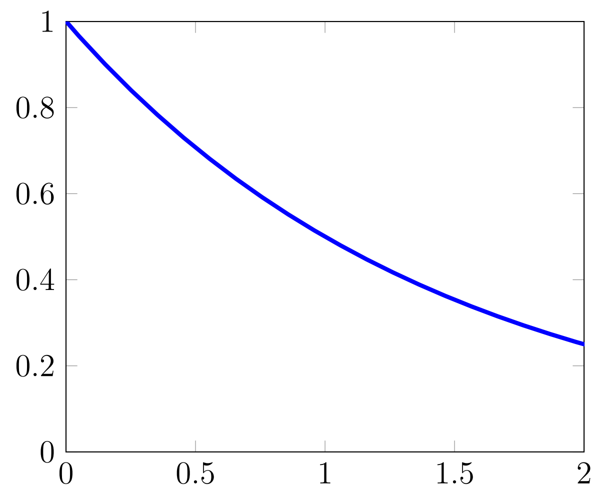
   with an error less than 10^{-7}?

2. Use the Midpoint rule to approximate the value of $\displaystyle\int_{-1}^{1} e^{-x^2}\,dx$ with $n = 4$. Draw a sketch to determine if the approximation is an overestimate or an underestimate of the integral.

3. The left, right, Trapezoidal, and Midpoint Rule approximations were used to estimate $\int_0^2 f(x)\,dx$, where f is the function whose graph is shown. The estimates were 0.7811, 0.8675, 0.8632, and 0.9540, and the same number of sub- intervals were used in each case.

   (a) Which rule produced which estimate?

   (b) Between which two approximations does the true value of $\int_0^2 f(x)\,dx$ lie?



4. Draw the graph of $f(x) = \sin\left(\frac{1}{2}x^2\right)$ in the viewing rectangle $[0, 1]$ by $[0, 0.5]$ and let $I = \int_0^1 f(x)\,dx$.

   (a) Use the graph to decide whether L_2, R_2, M_2, and T_2 underestimate or overestimate I.

   (b) For any value of n, list the numbers L_n, R_n, M_n, T_n, and I in increasing order.

   (c) Compute L_5, R_5, M_5, and T_5. From the graph, which do you think gives the best estimate of I?

5. The velocity in meters per second for a particle traveling along the axis is given in the table below. Use the Midpoint rule and Trapezoid rule to approximate the total distance the particle traveled from $t = 0$ to $t = 6$.

t	$v(t)$
0	0.75
1	1.34
2	1.5
3	1.9
4	2.5
5	3.2
6	3.0

Math Excel Worksheet # 5: Numerical Integration

1. The function f is continuous on the closed interval $[2, 14]$ and has values as show in the table below. Using three subintervals, what is the approximation of $\int_2^{14} f(x)dx$ using both the Midpoint and Trapezoid rule?

x	2	6	10	14
f(x)	12	28	34	30

2. (a) State Simpson's rule for approximating $\int_a^b f(x)\,dx$ using $N = 4$ intervals of size $\triangle x$.

   (b) The identity $\int_1^2 \frac{1}{x} dx = \ln(2)$ gives us a way to compute $\ln(2)$ using Simpson's rule. How many intervals N would be required to use Simpson's rule in order to compute $\ln(2)$ with an error of no more than 5×10^{-5}? Recall that the error estimate for Simpson's rule applied to $\int_a^b f(x)\,dx$ is $\text{Error}(S_N) \leq \frac{K_4(b-a)^5}{180N^4}$.