MA 114 Worksheet #27: Differential equations

1. (a) Is $y = \sin(3x) + 2e^{4x}$ a solution to the differential equation $y'' + 9y = 50e^{4x}$? Explain why or why not.

   (b) Explain why every solution of $dy/dx = y^2 + 6$ must be an increasing function.

   (c) What does is mean to say that a differential equation is linear or nonlinear?

2. Find all values of α so that $y(x) = e^{\alpha x}$ is a solution of the differential equation $y'' + y' - 12y = 0$.

3. A tank has pure water flowing into it at 10 liters/min. The contents of the tank are kept thoroughly mixed, and the contents flow out at 10 liters/min. Salt is added to the tank at the rate of 0.1 kg/min. Initially, the tank contains 10 kg of salt in 100 liters of water. Formulate an initial value problem (that is, a differential equation along with initial conditions) whose solution is the quantity of salt in the tank at any time t. Do not solve the initial value problem.

4. Consider a tank with 200 liters of salt-water solution. A salt-water solution, with a concentration of 2 grams per liter, pours into the tank at a rate of 4 liters per minute. The well-mixed solution in the tank pours out at the same rate of 4 liters/minute. Write a differential equation expressing the rate of change in the concentration, $c(t)$, of salt in the tank. Do not solve.

MA 114 MathExcel Worksheet # 27: Differential Equations and Direction Fields

1. Show that $y = \frac{2}{3}e^x + e^{-2x}$ is a solution of the differential equation $y' + 2y = 2e^x$.

2. Which of the following functions are solutions of the differential equation $y'' + y = \sin(x)$?

   (a) $y = \sin(x)$

   (b) $y = \frac{1}{2}x\sin(x)$

   (c) $y = \cos(x)$

   (d) $y = \frac{-1}{2}x\cos(x)$

3. A population is modeled by the differential equation $\dfrac{dP}{dt} = 1.2P\left(1 - \dfrac{P}{4200}\right)$.

   (a) For what values of P is the population increasing?

   (b) For what values of P is the population decreasing?

4. The Fitzhugh-Nagumo model for the electrical impulse in a neuron states that, in the absence of relaxation effects, the electrical potential in a neuron $v(t)$ obeys the differential equation

$$\frac{dv}{dt} = -v[v^2 - (1+a)v + a]$$

   where a is a positive constant such that $0 < a < 1$.

   (a) For what values of v is v unchanging (that is, $\dfrac{dv}{dt} = 0$)?

   (b) For what values of v is v increasing?

   (c) For what values of v is v decreasing?