MA 114 Worksheet #20: Arc length and surface area

1. (a) Write down the formula for the arc length of a function $f(x)$ over the interval $[a, b]$ including the required conditions on $f(x)$.

   (b) Write down the formula for the surface area of a solid of revolution generated by rotating a function $f(x)$ over the interval $[a, b]$ around the x-axis. Include the required conditions on $f(x)$.

2. Find an integral expression for the arc length of the following curves. Do ***not*** evaluate the integrals.

   (a) $f(x) = \sin(x)$ from $x = 0$ to $x = 2$.

   (b) $f(x) = x^4$ from $x = 2$ to $x = 6$.

   (c) $x^2 + y^2 = 1$

3. Find the arc length of the following curves.

   (a) $f(x) = x^{3/2}$ from $x = 100$ to $x = 101$.

   (b) $f(x) = \ln(\cos(x))$ from $x = 0$ to $x = \pi/3$.

   (c) $f(x) = e^x$ from $x = 0$ to $x = 1$.

4. Set up a function $s(t)$ that gives the arc length of the curve $f(x) = 2x + 1$ from $x = 0$ to $x = t$. Find $s(4)$.

5. Compute the surface areas of revolution about the x-axis over the given interval for the following functions.

   (a) $y = x$, $[0, 4]$

   (b) $y = x^3$, $[0, 2]$

   (c) $y = \left(4 - x^{2/3}\right)^{3/2}$, $[0, 8]$

   (d) $y = e^{-x}$, $[0, 1]$

   (e) $y = \sin x$, $[0, \pi]$

   (f) Find the surface area of the torus obtained by rotating the circle $x^2 + (y - b)^2 = r^2$ about the x-axis.

   (g) Show that the surface area of a right circular cone of radius r and height h is $\pi r\sqrt{r^2 + h^2}$.
   Hint: Rotate a line $y = mx$ about the x-axis for $0 \leq x \leq h$, where m is determined by the radius r.

MA 114 Math Excel Worksheet #20: Arc length and surface area

1. Set up the integral (but do not compute) that computes the surface area for a revolution about the x-axis over the given interval.
   (a) $y = (x+1)^2$ on $[0, 4]$
   (b) $y = e^{-x/3}$ on $[0, 1]$
   (c) $y = \sin(\pi x)$ on $[0, 1]$
2. Consider $f(x) = \frac{1}{4}x^2 - \frac{1}{2}\ln(x)$ on the interval $[1, e]$.
   (a) Calculate the arc length of $f(x)$ on $[1, e]$.
   (b) Calculate the volume of the solid of revolution obtained by rotating $f(x)$ about the x-axis on $[1, e]$.