MA 114 Worksheet #19: Volumes II

1. (a) Write a general integral to compute the volume of a solid obtained by rotating the region under $y = f(x)$ over the interval $[a, b]$ about the y-axis using the method of cylindrical shells.

   (b) If you use the disk method to compute the same volume, are you integrating with respect to x or y? Why?

2. Sketch the enclosed region and use the shell method to calculate the volume of the solid obtained by rotating the region about the y-axis.

   (a) $y = 3x - 2$, $y = 6 - x$, $x = 0$

   (b) $y = x^2$, $y = 8 - x^2$, $x = 0$, for $x \geq 0$

   (c) $y = 8 - x^3$, $y = 8 - 4x$, for $x \geq 0$

3. A soda glass has the shape of the surface generated by revolving the graph of $y = 6x^2$ for $0 \leq x \leq 1$ about the y-axis. Soda is extracted from the glass through a straw at the rate of 1/2 cubic inch per second. How fast is the soda level in the glass dropping when the level is 2 inches? (Answer should be implicitly in units of inches per second.)

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4. For each of the following, sketch the region bounded by the given curves and use the method of cylindrical shells to find the volume generated by rotating the region about the specified axis.

   (a) $y = x^3$, $y = 0$, $x = 1$, and $x = 2$; rotate about the y-axis.

   (b) $y = e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$; rotate about the y-axis.

   (c) $y = \sqrt{x}$, $x = 0$, $y = 2$; rotate about the x-axis.

   (d) $x = 1 + (y - 2)^2$, $x = 2$; rotate about the x-axis.

   (e) $y = 4x - x^2$, $y = 3$; rotate about $x = 1$.

5. Describe the solid whose volume is given by the integral $\int_0^1 2\pi(3 - y)(1 - y^2)\, dy$.

6. Use the shell method to find the volume of a sphere of radius r.

7. Recall that the torus is the solid obtained by rotating the circle $(x - a)^2 + y^2 = b^2$ around the y-axis (assume that $a > b$). Show that it has volume $2\pi^2 ab^2$ using the method of cylindrical shells.

8. Find the volume of a pyramid with height h and rectangular base with dimensions b and $2b$.

9. Find the volume common to two spheres, each with radius r, if the center of each sphere lies on the surface of the other sphere.