MA 114 Worksheet #17: Average value of a function

1. Write down the equation for the average value of an integrable function $f(x)$ on $[a, b]$.
2. Find the average value of the following functions over the given interval.
   (a) $f(x) = x^3,\ [0, 4]$
   (b) $f(x) = x^3,\ [-1, 1]$
   (c) $f(x) = \cos(x),\ \left[0, \frac{\pi}{6}\right]$
   (d) $f(x) = \frac{1}{x^2+1},\ [-1, 1]$
   (e) $f(x) = \frac{\sin(\pi/x)}{x^2},\ [1, 2]$
   (f) $f(x) = e^{-nx},\ [-1, 1]$
   (g) $f(x) = 2x^3 - 6x^2,\ [-1, 3]$
   (h) $f(x) = x^n$ for $n \geq 0$, $[0, 1]$
3. In a certain city the temperature (in $^\circ F$) t hours after 9 am was modeled by the function $T(t) = 50 + 14\sin\left(\frac{\pi t}{12}\right)$. Find the average temperature during the period from 9 am to 9 pm.
4. The velocity v of blood that flows in a blood vessel with radius R and length l at a distance r from the central axis is

$$v(r) = \frac{P}{4\eta l}(R^2 - r^2)$$

   where P is the pressure difference between the ends of the vessel and η is the viscosity of the blood. Find the average velocity (with respect to r) over the interval $0 < r < R$. Compare the average velocity with the maximum velocity.
5. Breathing is cyclic and a full respiratory cycle from the beginning of inhalation to the end of exhalation takes about 5 s. The maximum rate of air flow into the lungs is about 0.5 L/s. This explains, in part, why the function $f(t) = \frac{1}{2}\sin(2\pi t/5)$ has often been used to model the rate of air flow into the lungs. Use this model to find the volume of inhaled air in the lungs at time t. Then use this new function to compute the average volume of inhaled air in the lungs in one respiratory cycle.

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6. The linear density in a rod 8 m long is $12/\sqrt{x+1}$ kg/m, where x is measured in meters from one end of the rod. Find the average density of the rod.
7. If $f_{ave}[a, b]$ denotes the average value of f on the interval $[a, b]$ and $a < c < b$, show that

$$f_{ave}[a, b] = \left(\frac{c-a}{b-a}\right) f_{ave}[a, c] + \left(\frac{b-c}{b-a}\right) f_{ave}[c, b].$$