MA 114 Worksheet #13: The Ratio and Root Tests

1. (a) State the Root Test.
   (b) State the Ratio Test.

2. Identify the following statements as true or false. If the statement is true, cite evidence from the text to support it. If the statement is false, correct it so that it is a true statement from the text.
   (a) To prove that the series $\sum_{n=1}^{\infty} a_n$ converges you should compute the limit $\lim_{n\to\infty} a_n$. If this limit is 0 then the series converges.
   (b) To apply the Ratio Test to the series $\sum_{n=1}^{\infty} a_n$ you should compute $\lim_{n\to\infty} \frac{|a_{n+1}|}{|a_n|}$. If this limit is less than 1 then the series converges absolutely.
   (c) To apply the Root Test to the series $\sum_{n=1}^{\infty} a_n$ you should compute $\lim_{n\to\infty} \sqrt[n]{|a_n|}$. If this limit is 1 or larger then the series diverges.
   (d) One way to prove that a series is convergent is to prove that it is absolutely convergent.
   (e) An infinite series converges when the sequence of partial sums converges.

3. Determine whether the series is absolutely convergent, conditionally convergent, or divergent. Remember that you may use **any** tests you have learned.
   (a) $\sum_{n=0}^{\infty} \left(\frac{3n^3+2n}{4n^3+1}\right)^n$
   (b) $\sum_{n=1}^{\infty} 13\cos(5)^{n-1}$
   (c) $\sum_{n=1}^{\infty} \frac{2^n n^2}{n!}$
   (d) $\sum_{n=1}^{\infty} \frac{e^n}{n!}$
   (e) $\sum_{n=1}^{\infty} \frac{5^n}{(11-\cos^2(n))^n}$
   (f) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\sqrt[5]{n}}$
   (g) $\sum_{n=1}^{\infty} \frac{(-1)^n}{\ln(n+1)}$

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4. Use the Ratio Test to determine whether or not each of the following series converges, or state that the Ratio Test is inconclusive.
   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{5^n}$
   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{3^{n^2}}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{6^n}$

5. Use the Root Test to determine whether or not each of the following series converges, or state that the Root Test is inconclusive.
   (a) $\displaystyle\sum_{k=1}^{\infty} \left(\frac{k}{3k+10}\right)^k$
   (b) $\displaystyle\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{-n}$
   (c) $\displaystyle\sum_{n=1}^{\infty} \left(\frac{2n}{n+12}\right)^n$