MA 114 Worksheet #00: Review and Integration By Parts

1. Provide the most general antiderivative of the following functions:
   (a) $f(x) = x^4 + x^2 + x + 1000$
   (b) $g(x) = (3x - 2)^{20}$
   (c) $h(x) = \dfrac{\sin(\ln(x))}{x}$

2. Compute the following definite integrals:
   (a) $\displaystyle\int_{-1}^{1} e^{u+1}\,du$
   (b) $\displaystyle\int_{-2}^{2} \sqrt{4 - x^2}\,dx$
   (c) $\displaystyle\int_{1}^{9} \frac{x-1}{\sqrt{x}}\,dx$
   (d) $\displaystyle\int_{0}^{10} |x - 5|\,dx$
   (e) $\displaystyle\int_{0}^{1} xe^{-x^2}\,dx$

   Hint: For some of the integrals, you will need to interpret the integral as an area and use facts from geometry to compute the integral.

3. Write as a single integral in the form $\int_a^b f(x)\,dx$:

$$\int_{-2}^{2} f(x)\,dx + \int_{2}^{5} f(x)\,dx - \int_{-2}^{-1} f(x)\,dx$$

4. Evaluate the following:
   (a) $\displaystyle\int_{0}^{4} (3x^{0.5} - 2xe^{-x^2})\,dx$
   (b) $\displaystyle\int_{0}^{1} \frac{e^{2x}}{1 + e^{2x}}\,dx$
   (c) $\displaystyle\int \frac{[\ln(s)]^2}{s}\,ds$
   (d) $\displaystyle\int (z^3 + 1)\sin(z^4 + 4z)\,dz$
   (e) $G'(2)$, if $G(x) = \displaystyle\int_{1}^{x^3} te^t\,dt$
   (f) $A'(x)$, if $A(x) = \displaystyle\int_{2}^{\sqrt{3x}} \sin(t)\,dt$
   (g) $\displaystyle\int_{-2}^{1} 3 + 2|x|\,dx$

5. Use calculus to find the area of the triangle with the vertices $(2, 0), (0, 2)$, and $(-1, 1)$.

6. Evaluate the integral

$$\int_{0}^{2\pi} \sqrt{1 - \cos^2(x)}\,dx.$$

7. Find $\int_0^2 f(x)\,dx$, where

$$f(x) = \begin{cases} x^2 & \text{if } 0 \le x \le 1 \\ x & \text{if } 1 < x \le 2 \end{cases}.$$

8. Evaluate the following integrals using integration by parts.
   (a) $\int x^2 \ln(x) dx$ *Hint:* Let $u = \ln(x)$ and $dv = x^2 dx$.
   (b) $\int \theta \cos(\theta) d\theta$ *Hint:* Let $u = \theta$ and $dv = \cos(\theta) d\theta$.
   (c) $\int x \cos(5x) dx$
   (d) $\int t e^{-3t} dt$
   (e) $\int (x-1)\sin(\pi x) dx$
   (f) $\int (x^2 + 2x)\cos(x) dx$