Trig Identities and Integration Strategies

Pythagorean Identities:

- $\sin^2(x) + \cos^2(x) = 1$
- $\tan^2(x) + 1 = \sec^2(x)$
- $1 + \cot^2(x) = \csc^2(x)$

Note: The second two identities can be derived from the first one by dividing through by $\cos^2(x)$ and $\sin^2(x)$, respectively.

Double-Angle Formulas:

- $\sin(2x) = 2\sin(x)\cos(x)$
- $\cos(2x) = \cos^2(x) - \sin^2(x)$

Power Reduction Formulas:

- $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$
- $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$

Note: These are good for integrating even powers of sin and cos when making a substitution is inconvenient.

Integration Strategies:

- **Substitution:** Use to integrate composite functions of the form $\int f(g(x)) \cdot g'(x)\, dx$. Make the substitution $u = g(x)$ and $du = g'(x)dx$.
- **Integration by Parts:** Use to integrate products of functions $\int f(x) \cdot g(x)\, dx$. Let u be the function you'd like to differentiate (e.g. a polynomial whose degree will decrease or a trig function that's hard to integrate) and dv be the function you'd rather integrate. Then evaluate using the formula $\int u\, dv = uv - \int v\, du$.
- **Trig Substitution:** Use when you have a function whose structure could be simplified by one of the trig identities above, e.g., $\sqrt{a - bx^2}$. Substitute the appropriate trig function for x and be sure to replace dx as well.
- **Partial Fractions:** Use to integrate fractions whose denominator can be factored. Rewrite as a sum or difference of fractions that are easier to integrate (using long division if the numerator has higher degree than the denominator).

Also:

- Don't forget to change back to the original variable and add C on indefinite integrals!
- Remember to change limits or denote different variable limits and switch variables back on definite integrals!