Worksheet # 8: Review for Exam I

1. Find all real values of the constants a and b for which the function $f(x) = ax + b$ satisfies:

   (a) $f \circ f(x) = f(x)$ for all x.

   (b) $f \circ f(x) = x$ for all x.

2. Simplify the following expressions:

   (a) $\log_5(125)$

   (b) $(\log_4(16))(\log_4(2))$

   (c) $\log_x(x(\log_y(y^x)))$

   (d) $\log_\pi(1 - \cos(x)) + \log_\pi(1 + \cos(x)) - 2\log_\pi \sin(x)$

3. Suppose that $\tan(x) = \frac{3}{4}$ and $-\pi < x < 0$. Find $\cos(x)$, $\sin(x)$, and $\sin(2x)$.

4. (a) Solve the equation $3^{2x+5} = 4$ for x. Show each step in your computation.

   (b) Express the quantity $\log_2(x^3 - 2) + \dfrac{1}{3}\log_2(x) - \log_2(5x)$ as a single logarithm. For which values of x can we compute this quantity?

5. Suppose that the height of an object at time t is given by $h(t) = 5t^2 + 40t$.

   (a) Find the average velocity of the object on the interval $[a, a + h]$.

   (b) Find the average velocity of the object on the intervals $[2.9, 3]$, $[2.99, 3]$, $[2.999, 3]$, $[3, 3.001]$, $[3, 3.01]$, and $[3, 3.1]$.

   (c) Use your answer from part (b) to estimate the instantaneous velocity at $t = 3$.

6. Calculate the following limits using the limit laws. Carefully show your work!

   (a) $\lim_{x\to 0}(2x - 1)$

   (b) $\lim_{x\to 0} \dfrac{\sqrt{x+4} - 2}{x}$

7. For each of the following limits, calculate the limit or explain why it does not exist.

   (a) $\lim_{x\to 1} \dfrac{x - 2}{\frac{1}{x} - \frac{1}{2}}$

   (b) $\lim_{x\to 2} \dfrac{x - 2}{\frac{1}{x} - \frac{1}{2}}$

   (c) $\lim_{x\to 2^+} \dfrac{x^2 - 1}{x - 2}$

   (d) $\lim_{x\to a}(xa - a^2)$

   (e) $\lim_{x\to 4} \dfrac{\sqrt{x} - 2}{x^2 - 16}$

   (f) $\lim_{x\to 2} \dfrac{x + 1}{x - 2}$

   (g) $\lim_{x\to 2} \dfrac{x^3 - 8}{x - 2}$

   (h) $\lim_{a\to x}(xa - a^2)$

8. (a) State the Squeeze Theorem.

   (b) Use the Squeeze Theorem to find the following limits:

      i. $\lim_{x\to 0} x \sin \dfrac{1}{x^2}$

      ii. $\lim_{x\to \frac{\pi}{2}} \cos x \cos(\tan x)$

9. Suppose $f(x) = \dfrac{|x-3|}{x^2 - x - 6}$. Find the following limits:

(a) $\displaystyle\lim_{x\to 3^+} f(x)$

(b) $\displaystyle\lim_{x\to 3^-} f(x)$

(c) $\displaystyle\lim_{x\to 3} f(x)$

10. Suppose $\displaystyle\lim_{x\to 2} f(x) = 3$ and $\displaystyle\lim_{x\to 2} g(x) = 5$. For each of the following limits, find the limit or explain why you need more information.

(a) $\displaystyle\lim_{x\to 2}(2f(x) + 3g(x))$

(b) $\displaystyle\lim_{x\to 2} \frac{f(x)}{g(x)+1}$

(c) $\displaystyle\lim_{x\to 2} f(2)g(x)$

(d) $\displaystyle\lim_{x\to 2} \frac{x-2}{2f(x)-6}$

11. (a) State the definition of the continuity of a function $f(x)$ at the point $x = a$.

(b) Find the constant a so that the following function is continuous everywhere.

$$f(x) = \begin{cases} \dfrac{x^2 - a^2}{x - a} & \text{if } x \neq a \\ 8 & \text{if } x = a \end{cases}$$

12. If $g(x) = x^2 + 5^x - 3$, use the Intermediate Value Theorem to show that there is a number a such that $g(a) = 10$.

13. Complete the following statements:

(a) A function $f(x)$ passes the horizontal line test if the function f is ______________________

(b) If $\displaystyle\lim_{x\to a} f(x)$ and $\displaystyle\lim_{x\to a} g(x)$ exist, then ______________________ guarantees that

$$\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$$

(c) $\displaystyle\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = L$ if and only if ______________________________________

(d) Let $g(x) = \begin{cases} x & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$ be a piecewise function.
The function $g(x)$ is NOT continuous at $x =$__________ since ______________________

(e) Let $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ 1 & \text{if } x = 0 \\ x & \text{if } x > 0 \end{cases}$ be a piecewise function.
The function $f(x)$ is NOT continuous at $x =$__________ since ______________________

Supplemental Worksheet #8: Exam Review

1. Inverse Functions

(a) Find the largest value c such that $f(x) = (x+2)^2 + 13$ is one to one on the interval $(-\infty, c]$.

(b) Restrict the domain of f to $(-\infty, c]$ and find the formula for the inverse function. Call it g.

(c) Give the domain and range of g.

2. Determine if the following statements are true or false. If true, explain why. If false, provide a counterexample.

   (a) If $\lim_{x\to 3^+} f(x) = 4$ and f is continuous at 3, then $f(3) = 4$.

   (b) If $\lim_{x\to 4^-} f(x) = \lim_{x\to 4^+} f(x) = 3$, then f is continuous at 4.

   (c) If p is a polynomial, then $\lim_{x\to b} p(x) = p(b)$.

   (d) If $\lim_{x\to 1} f(x) = \lim_{x\to 1} g(x)$, then $\lim_{x\to 1^+} f(x) = \lim_{x\to 1^-} g(x)$

   (e) If $f(2) = g(2)$, then $\lim_{x\to 2} f(x) = \lim_{x\to 2} g(x)$.

3. Use the Intermediate Value theorem to show that $e^{-x^3} = x^2$ has a solution on $(0, 1)$.