Worksheet # 5: Limits: A Numerical and Graphical Approach

1. For each task or question below, provide a specific example of a function $f(x)$ that supports your answer.

   (a) In words, briefly describe what "$\lim_{x\to a} f(x) = L$" means.

   (b) In words, briefly describe what "$\lim_{x\to a} f(x) = \infty$" means.

   (c) Suppose $\lim_{x\to 1} f(x) = 2$. Does this imply $f(1) = 2$?

   (d) Suppose $f(1) = 2$. Does this imply $\lim_{x\to 1} f(x) = 2$?

2. Compute the value of the following functions near the given $x-$value. Use this information to guess the value of the limit of the function (if it exists) as x approaches the given value.

   (a) $f(x) = 2^{x-1} + 3$, $x = 1$

   (b) $f(x) = \frac{\sin(2x)}{x}$, $x = 0$

   (c) $f(x) = \sin\left(\frac{\pi}{x}\right)$, $x = 0$

   (d) $f(x) = \frac{x^2-3x+2}{x^2+x-6}$, $x = 2$

3. Let $f(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ x-1 & \text{if } 0 < x \text{ and } x \neq 2 \\ -3 & \text{if } x = 2 \end{cases}$.

   (a) Sketch the graph of f.

   (b) Compute the following:

      i. $\lim_{x\to 0^-} f(x)$

      ii. $\lim_{x\to 0^+} f(x)$

      iii. $\lim_{x\to 0} f(x)$

      iv. $f(0)$

      v. $\lim_{x\to 2^-} f(x)$

      vi. $\lim_{x\to 2^+} f(x)$

      vii. $\lim_{x\to 2} f(x)$

      viii. $f(2)$

4. In the following, sketch the functions and use the sketch to compute the limit.

   (a) $\lim_{x\to \pi} x$

   (b) $\lim_{x\to 3} \pi$

   (c) $\lim_{x\to a} |x|$

   (d) $\lim_{x\to 3} 2^x$

5. Show that $\lim_{h\to 0} \frac{|h|}{h}$ does not exist by examining one-sided limits. Then sketch the graph of $\frac{|h|}{h}$ to verify your reasoning.

6. Compute the following limits or explain why they fail to exist.

   (a) $\lim_{x\to -3^+} \frac{x+2}{x+3}$

   (b) $\lim_{x\to -3^-} \frac{x+2}{x+3}$

   (c) $\lim_{x\to -3} \frac{x+2}{x+3}$

   (d) $\lim_{x\to 0^-} \frac{1}{x^3}$

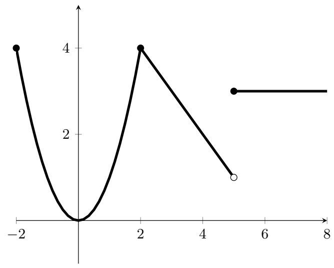
7. In the theory of relativity, the mass of a particle with velocity v is:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

   where m_0 is the mass of the particle at rest and c is the speed of light. What happens as $v \to c^-$?

Supplemental Worksheet # 5: Limits: A Numerical and Graphical Approach

1. Use the graph of $f(x)$ to evaluate the following limits.



(a) $\lim_{x \to 2^-} f(x)$ (b) $\lim_{x \to 2^+} f(x)$ (c) $\lim_{x \to 2} f(x)$

(e) $\lim_{x \to 5^-} f(x)$ (f) $\lim_{x \to 5^+} f(x)$ (f) $\lim_{x \to 5} f(x)$

2. Sketch the graph of a function with the given limits.

(a) $\lim_{x \to 1} f(x) = 0$, $\lim_{x \to 3^+} f(x) = 3$, $\lim_{x \to -1} f(x) = \infty$

(b) $\lim_{x \to 2^-} f(x) = \infty$, $\lim_{x \to 2^+} f(x) = -\infty$, $\lim_{x \to -1} f(x) = 0$

(c) $\lim_{x \to 0} f(x) = 2$, $\lim_{x \to 2} f(x) = -1 \neq f(2)$, $\lim_{x \to 3} f(x) = -1$