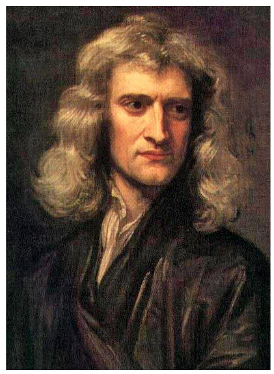
Worksheet # 4: Average and Instantaneous Velocity

An Interesting Fact: Isaac Newton is one of the two people usually credited with discovering calculus, the other being Gottfried Leibniz, who was his contemporary. They both worked in the 1660's and 1670's. Newton was highly motivated by problems in physics and engineering, and his language for calculus reflected this. When studying average and instantaneous velocity, Newton referred to the time variable as a *fluent* and the velocity of the object as the *fluxion*. Newton would refer to an infinitely small passage of time as a *moment*, and calculate fluxions for moments of fluents.

Newton's success was in part the result of his intensity — when he was particularly interested in a problem, Newton would neglect to eat or sleep for long periods of complete focus on the problem. He would go to phenomenal lengths to satisfy his curiosity: once as a college student, he stuck a stick in his eye and twisted it around to observe what effect it had on his vision. In 1693 he had a nervous breakdown, often attributed to his habit of tasting chemicals in his laboratory, as part of his alchemy experiments. (Newton spent half of his life attempting to create the "philosopher's stone"... if only he had attended Hogwarts.)



1. A ball is thrown vertically into the air from ground level with an initial velocity of 15 m/s. Its height at time t is $h(t) = 15t - 4.9t^2$.
   (a) How far does the ball travel during the time interval $[1, 3]$?
   (b) Compute the ball's average velocity over the time interval $[1, 3]$.
   (c) Graph the curve $y = h(t)$ and the line between the points $(1, h(1))$ and $(3, h(3))$. How does the slope of this line relate to your answer in part (b)?
   (d) Compute the ball's average velocity over the time intervals $[1, 1.01]$, $[1, 1.001]$,$[0.99, 1]$, and $[0.999, 1]$.
   (e) Estimate the instantaneous velocity when $t = 1$.
2. A particle moves along a line and its position $p(t)$ in meters after time t seconds is given by the following table.

t	0.0	0.2	0.5	0.65	0.9	1.1	1.15	1.3
$p(t)$	3	4.2	5.7	8.8	7.6	8.0	9.0	9.5

   (a) Describe the motion of the particle between 0 seconds and 1.3 seconds. Justify your description by representing the table as points $(t, p(t))$ plotted in the plane.
   (b) Consider the average velocity of the particle across different time intervals. Based on your calculations, can you conclude at what point in time the particle is moving with the largest positive *instantaneous* velocity? Why or why not? Based on this data, when do you believe that the particle is likely to be moving with the largest positive instantaneous velocity? Why?

3. With the other members of your group, compare and contrast the first two problems on this worksheet.
   (a) Which of these two problems do you believe is most representative of the type of problem a scientist or engineer will encounter when analyzing a real-world experiment? Why?
   (b) Which of these two problems do you believe is most representative of the type of problem a scientist or engineer will encounter when setting up a mathematical model of a physical situation? Why?
4. Let $p(t) = t^3 - 45t$ denote the distance (in meters) to the right of the origin of a particle at time t minutes after noon.
   (a) Find the average velocity of the particle on the intervals $[2, 2.1]$ and $[2, 2.01]$.
   (b) Use this information to guess a value for the instantaneous velocity of particle at 12:02pm.
5. A particle moves along a line and its position after time t seconds is $p(t) = 3t^3 + 2t$ meters to the right of the origin. Approximate the instantaneous velocity of the particle at $t = 2$.
6. A particle is moving along a straight line so that its position at time t seconds is given by $s(t) = 4t^2 - t$ meters.
   (a) Find the average velocity of the particle over the time interval $[1, 2]$.
   (b) Determine the average velocity of the particle over the time interval $[2, t]$ where $t > 2$. Simplify your answer. [Hint: Factor the numerator.]
   (c) Based on your answer in (b) can you guess a value for the instantaneous velocity of the particle at $t = 2$?

Supplemental Worksheet # 4: Average and Instantaneous Velocity

1. A particle moves along a straight line and its position after x seconds is given by $f(x)$ (in feet). Calculate the average velocity of a particle of the following position functions on the given interval.
   (a) $f(x) = x^2 - x + 2$, $[-1, 5]$
   (b) $f(x) = x^{\frac{1}{3}} - 2$, $[1, 8]$
   (c) $f(x) = \frac{x}{x-1}$, [2,3]
2. A rock is dropped off of a bridge and its height at time t is $h(t) = -4.9t^2 + 120$ meters.
   (a) Evaluate the rock's average velocity over the time intervals $[1, 1.001]$, $[1, 1.0001]$, $[.999, 1]$, and $[.9999, 1]$.
   (b) Use your answers to estimate the instantaneous velocity at $t = 1$.
3. Suppose an object moves along the y-axis with so that its location is $y = x^2 - 12x + 1$ (here x is in minutes and y is in feet).
   (a) Find the average velocity of the object over the time interval $[3, 10]$.
   (b) Find the average velocity of the object for x changing from 6 to $6 + h$ minutes.