Worksheet # 28: Linear and Higher-Order Approximation and Applications

1. What is the relation between the linearization of a function $f(x)$ at $x = a$ and the tangent line to the graph of the function $f(x)$ at $x = a$ on the graph?

2. (a) Use the linearization of $\sqrt{x}$ at $a = 16$ to estimate $\sqrt{18}$.
   (b) Find a decimal approximation to $\sqrt{18}$ using a calculator.
   (c) Compute both the absolute error and the percentage error if we use the linearization to approximate $\sqrt{18}$.

3. For each of the following, use a linear approximation to the change in the function and a convenient nearby point to estimate the value:
   (a) $(3.01)^3$
   (b) $\sqrt{17}$
   (c) $8.06^{2/3}$

4. Suppose we want to paint a sphere of radius 200 cm with a coat of paint 0.1 mm thick. Use a linear approximation to approximate the amount of paint we need to do the job.

5. Let $f(x) = \sqrt{16+x}$. First, find the linearization to $f(x)$ at $x = 0$, then use the linearization to estimate $\sqrt{15.75}$. Present your solution as a rational number.

6. Find the linearization $L(x)$ to the function $f(x) = \sqrt{1-2x}$ at $x = -4$.

7. Find the linearization $L(x)$ to the function $f(x) = \sqrt[3]{x+4}$ at $x = 4$, then use the linearization to estimate $\sqrt[3]{8.25}$.

8. Your physics professor tells you that you can replace $\sin(\theta)$ with θ when θ is close to zero. Explain why this is reasonable.

9. Suppose we measure the radius of a sphere as 10 cm with an accuracy of $\pm$ 0.2 cm. Use linear approximations to estimate the maximum error in:
   (a) the computed surface area.
   (b) the computed volume.

10. Suppose that $y = y(x)$ is a differentiable function which is defined near $x = 2$, satisfies $y(2) = -1$ and
$$x^2 + 3xy^2 + y^3 = 9.$$
Use the linear approximation to the change in y to approximate the value of $y(1.91)$.

11. Use Taylor polynomials with $a = 0$ to approximate $\frac{1}{\sqrt[10]{e}}$ to five decimal places.

12. (a) Use Taylor polynomials with $a = 0$ to approximate $\int_0^1 \sin(x^4)\,dx$ to four decimal places.
    (b) Can you find an indefinite integral for this integrand? Why or why not?

13. Use Taylor polynomials with $a = 0$ to approximate $\cos(1)$ to four decimal places.

14. (a) Use Taylor polynomials with $a = 0$ to approximate $\int_0^{0.5} x^2 e^{-x^2}\,dx$ to two decimal places.
    (b) Can you find an indefinite integral for this integrand? Why or why not?

15. If $f(x) = (1+x^3)^{30}$, what is $f^{(58)}(0)$?

MathExcel Worksheet # 28 Supplemental Problems

16. Suppose that a curve is given by the equation $x^2 + y^3 = 2x^2y$. Verify that the point (1, 1) lies on the curve. Use linear approximation to estimate the value of the y-coordinate when $x = 1.2$.

17. A function $f(x)$ is approximated near $x = 0$ by the second-degree Taylor polynomial $T_2(x) = 5 - 7x + 8x^2$. Find the values of $f(0)$, $f'(0)$, $f''(0)$, and $f'''(0)$, if possible.

18. (a) Find a parabola to best approximate the unit circle $x^2 + y^2 = 1$ near the point $(0, 1)$.

    (b) Use your answer to part (a) to estimate the y-coordinate of the point on the upper half of the unit circle with the x-coordinate equal to 0.1.