Worksheet # 26: The Fundamental Theorem of Calculus and Net Change

1. (a) State both parts of the Fundamental Theorem of Calculus using complete sentences.

   (b) Consider the function $f(x)$ on $[1, \infty)$ defined by $f(x) = \displaystyle\int_1^x \sqrt{t^5 - 1}\, dt$. Find the derivative of f. Explain why the function f is increasing.

   (c) Find the derivative of the function $g(x) = \int_1^{x^3} \sqrt{t^5 - 1}\, dt$ on $(1, \infty)$.

2. Use Part 2 of the Fundamental Theorem of Calculus to evaluate the following integrals or explain why the theorem does not apply:

   (a) $\displaystyle\int_{-2}^{5} 6x\, dx$

   (b) $\displaystyle\int_{-2}^{7} \frac{1}{x^5}\, dx$

   (c) $\displaystyle\int_{-1}^{1} e^{u+1}\, du$

   (d) $\displaystyle\int_{\frac{\pi}{3}}^{\frac{\pi}{6}} \frac{\sin(2x)}{\sin(x)}\, dx$

3. Find each of the following indefinite integrals.

   (a) $\displaystyle\int 7x - 2\, dx$

   (b) $\displaystyle\int \frac{1}{x^{78}}\, dx$

   (c) $\displaystyle\int e^{u+2}\, du$

4. A population of rabbits at time t increases at a rate of $40 - 12t + 3t^2$ rabbits per year where t is measured in years. Find the population after 8 years if there are 10 rabbits at $t = 0$.

5. Suppose the velocity of a particle traveling along the x-axis is given by $v(t) = 3t^2 + 8t + 15$ m/s at time t seconds. The particle is initially located 5 meters left of the origin. How far does the particle travel from $t = 2$ seconds to $t = 3$ seconds? After 3 seconds, where is the particle with respect to the origin?

6. Suppose an object traveling in a straight line has a velocity function given by $v(t) = t^2 - 8t + 15$ km/hr. Find the displacement and distance traveled by the object from $t = 2$ to $t = 4$ hours.

7. (a) An oil storage tank ruptures and oil leaks from the tank at a rate of $r(t) = 100e^t$ liters per minute. How much oil leaks out during the first hour?

   (b) Is this model realistic? In other words, is it realistic to use this function $r(t)$ to model the leak rate in this situation? Why or why not?

8. Recognize each of the sums as a Riemann sum, express the limit as an integral and use the Fundamental Theorem to evaluate the limit.

   (a) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\sqrt{3 + \frac{i}{n}}}{n}$

   (b) $\displaystyle\lim_{n\to\infty} \sum_{i=1}^{n} 2\frac{(2 + \frac{2i}{n})^2}{n}$

MathExcel Worksheet # 26: FTC and the Net Change Theorem

9. Evaluate the following:

   (a) $\displaystyle\int_{e^a}^{e^b} \frac{1}{t} dt$

   (b) $\displaystyle\int_0^x -5t^4 + \frac{1}{5t+4} dt$

   (c) $\displaystyle\frac{d}{dx}\int_0^x -5t^4 + \frac{1}{5t+4} dt$

10. Suppose $f(t)$ is a continuous function and suppose that

$$\int_0^x f(t)dt = xe^{2x} + \int_0^x e^{-t} f(t)dt.$$

    Determine $f(t)$. *Hint: Differentiate both sides.*

11. Conisder the following:

$$\int_{-3}^{2} \frac{1}{x^2} dx = -x^{-1}\Big|_{-3}^{2} = (-\frac{1}{2}) - (-\frac{1}{-3}) = -\frac{1}{2} - \frac{1}{3} = \frac{-5}{6}$$

    What is wrong with this calculation?

12. Evaluate the following integrals by interpreting them as geometric areas.

   (a) $\displaystyle\int_{-7}^{7} \sqrt{49 - x^2} dx$

   (b) $\displaystyle\int_{-7}^{12} |x+2| dx$

   (c) $\displaystyle\int_4^{15} f(x)dx$ where $f(x) = \begin{cases} x & x \le 9 \\ 12 & 9 < x < 13 \\ -x + 15 & 13 \le x \end{cases}$