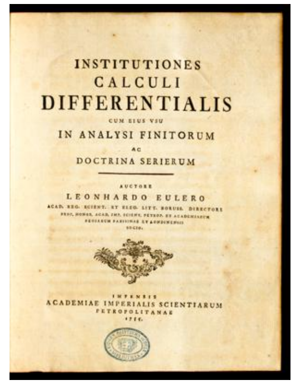
Worksheet # 25: The Fundamental Theorem of Calculus, Part 1

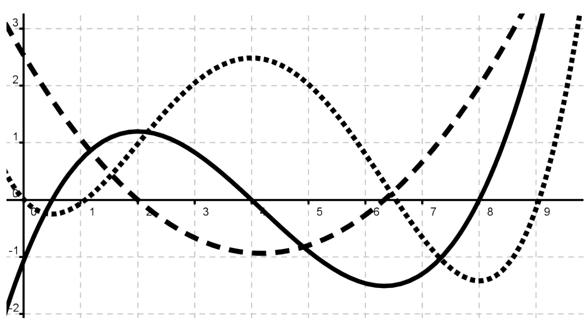


INSTITUTIONES
CALCULI
DIFFERENTIALIS
IN ANALYSI FINITORUM
DOCTRINA SERIERUM
LEONHARDO EULERO
ACADEMIAE IMPERIALIS SCIENTIARUM
PETROPOLITANAE

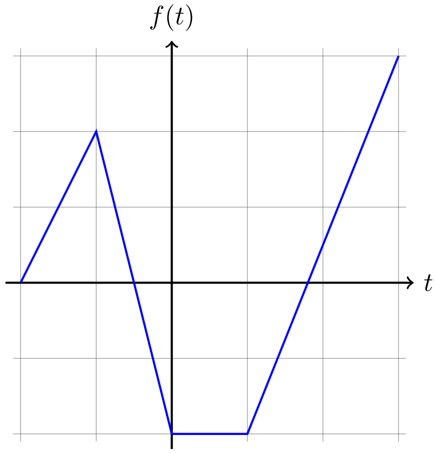
An Interesting Fact: One of the most influential mathematicians of all time is Leonhard Euler, who lived from 1707 to 1783. He worked in every branch of mathematics, and also in physics, astronomy, engineering, and music theory. Euler (pronounced "OIL-ER") was so productive in so many areas, it has been estimated that he wrote roughly one-third of the research publications in the mathematical sciences in Europe between 1725 and 1800.

In addition to his research work, Euler wrote several textbooks such as *Institutiones calculi differentialis* (1755) and *Institutionum calculi integralis* (1768–1770) (Foundations of Differential and Integral Calculus). Because Euler's textbooks were so influential, his notation and conventions are what we use today, for example writing $f(x)$ for functions, π for the ratio of the circumference to diameter of a circle, and e for the base of the natural logarithm.

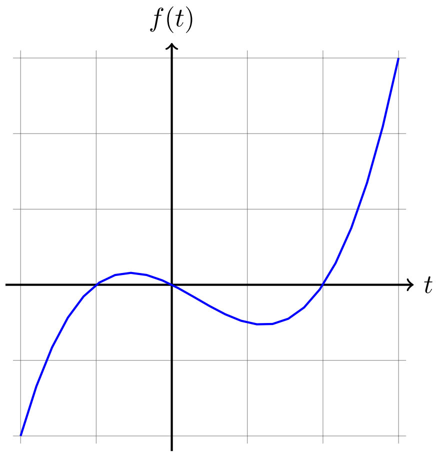
1. Below is pictured the graph of the function $f(x)$, its derivative $f'(x)$, and the integral $\int_0^x f(t)\,dt$. Identify $f(x), f'(x)$ and $\int_0^x f(t)\,dt$ and explain your reasoning.



2. Let $g(x) = \int_{-2}^x f(t)\,dt$ where f is the function whose graph is shown below.
   (a) Evaluate $g(-1)$, $g(0)$, $g(1)$, $g(2)$, and $g(3)$.
   (b) On what interval is g increasing? Why?
   (c) Where does g have a maximum value? Why?



3. Let $g(x) = \int_{-2}^{x} f(t)\,dt$ where f is the function whose graph is shown below. Where is $g(x)$ increasing and decreasing? Explain your answer.



4. Let $F(x) = \int_{2}^{x} e^{t^2}\,dt$. Find an equation of the tangent line to the curve $y = F(x)$ at the point with x-coordinate 2.

5. Use Part 1 of the Fundamental Theorem of Calculus to find the derivative of the following functions:

   (a) $g(x) = \displaystyle\int_1^x \left(2+t^4\right)^5\,dt$

   (b) $F(x) = \displaystyle\int_x^4 \cos\left(t^5\right)\,dt$

   (c) $h(x) = \displaystyle\int_0^{x^2} \sqrt[3]{1+r^3}\,dr$

   (d) $y(x) = \displaystyle\int_{\frac{1}{x^2}}^{0} \sin^3(t)\,dt$

   (e) $G(x) = \displaystyle\int_{\sqrt{x}}^{x^2} \sqrt{t}\sin(t)\,dt$

6. Find a function $f(t)$ and a number a such that

$$6 + \int_a^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x}$$

for all $x > 0$.

Math Excel Worksheet Supplementary Problems # 25

1. Evaluate the integral by interpreting it as an area:

$$\int_{-\sqrt{2}}^{\sqrt{2}} \sqrt{2-x^2}dx.$$

2. Using geometry and the fact that $\int_0^a x^2dx = \frac{a^3}{3}$, evaluate the integral:

$$\int_0^1 (x^2+\sqrt{1-x^2})dx.$$

3. Using tools from geometry, prove that:

$$\int_0^a xdx = \frac{1}{2}a^2.$$