Worksheet # 24: Review for Exam III

1. Strontium-90 has a half-life of 28 days.
   (a) A sample of strontium-90 has an initial mass of 50 mg. Find a formula for the mass remaining after t days.
   (b) Find the mass remaining after 40 days.
   (c) How long does it take the sample to decay to a mass of 2 mg?
2. Describe in words and diagrams how to use the first and second derivative tests to identify and classify extrema of a function $f(x)$.
3. Find the absolute minimum of the function $f(t) = t + \sqrt{1 - t^2}$ on the interval $[-1, 1]$. Be sure to specify the value of t where the minimum is attained and justify your answer.
4. (a) Consider the function $f(x) = 2x^3 + 3x^2 - 72x - 47$ on $(-\infty, \infty)$.
      i. Find the critical number(s) of f.
      ii. Find the intervals on which f is increasing or decreasing.
      iii. Find the local maximum and minimum values of f.
      iv. Find the inflection points and the intervals of concavity of f.
   (b) Repeat with the function $f(x) = x^4 - 2x^2 + 3$.
   (c) Repeat with the function $f(x) = e^{2x} + e^{-x}$.
5. For what values of c does the polynomial $p(x) = x^4 + cx^3 + x^2$ have two inflection points? One inflection point? No inflection points?
6. (a) State the Mean Value Theorem. Use complete sentences.
   (b) Does there exist a function f such that $f(0) = -1, f(2) = 4$, and $f'(x) \leq 2$ for all x?
7. State L'Hopital's Rule for limits in indeterminate form of type 0/0. Use complete sentences, and include all necessary assumptions. Then evaluate the following limits:
   (a) $\displaystyle\lim_{x\to 0} \frac{e^x - x - 1}{x^2}$
   (b) $\displaystyle\lim_{x\to 0^+} x^3 \ln(x)$
   (c) $\displaystyle\lim_{x\to -\infty} \frac{x+2}{\sqrt{9x^2+1}}$
   (d) $\displaystyle\lim_{x\to 2} \frac{e^{2x}}{x+2}$
8. A poster is to have an area of 180 cm^2 with 1 cm margins at the bottom and sides and 2 cm margins at the top. What dimensions will give the largest printed area? Be sure to explain how you know you have found the largest area.
   (a) Draw a picture and write the constraint equation.
   (b) Write the function you are asked to maximize or minimize and determine its domain.
   (c) Find the maximum or minimum of the function that you found in part (b).
9. Find a positive number such that the sum of the number and twice its reciprocal is small as possible.
10. Find the most general antiderivative for each of the following:
   (a) $f(x) = 5x^{10} + 7x^2 + x + 1$
   (b) $g(x) = x + \cos(2x + 1)$

(c) $h(x) = \dfrac{1}{x+1}$, where $x + 1 > 0$

11. Find a function with $f''(x) = \sin(2x)$, $f(\pi) = 1$, and $f'(0) = 2$.

12. Consider the region bounded by the graph of $f(x) = \frac{1}{x}$, the x-axis, and the lines $x = 1$ and $x = 2$. Find L_3, the left endpoint approximation of this area with 3 subdivisions.

13. Suppose we know that $\sum_{k=1}^{n} a_k = n^2 + 2n$. Using this information, find the following:

    (a) $\sum_{k=1}^{20} (4a_k + 1)$.

    (b) $\sum_{k=5}^{10} a_k$.

14. (a) Let $f(x) = (x-4)^2$. Without finding c, use the Mean Value Theorem to show that there is a number c in the interval $(3, 5)$ such that $f'(c) = 0$.

    (b) Let $g(x) = (x-4)^{-2}$. Show that there is no value of c in the interval $(3, 5)$ such that $g(5) - g(3) = g'(c)(5-3)$, and explain why this does not contradict the Mean Value Theorem.

Math Excel Worksheet Supplementary Problems # 24

1. If $f(2) = 30$ and $f'(x) \geq 4$ for $2 \leq x \leq 6$, how small can $f(6)$ be?
2. Suppose a sculptor can sell 15 statues at \$500 each, but for each additional statue she makes, the price goes down by \$15 (they are becoming less trendy). How many statues should she produce to maximize her revenue? What is her maximum revenue?
3. Consider the function $f(x) = x^2 + 3$. We are interested in the area A under the graph of $f(x)$ on the interval $[1, 5]$.

    (a) Divide the interval $[1, 5]$ into n subintervals of equal length and write an expression for R_n, the sum that represents the right-endpoint approximation of the area A.

    (b) Use the formula $\sum_{i=1}^{N} i^2 = \dfrac{N(N+1)(2N+1)}{6}$ to find a closed expression for R_n.

    (c) Take the appropriate limit of R_n to find an exact value for the area A.
4. Suppose that an object is fired downward, with an unknown velocity, from a plane flying at 10,700 m. If the object strikes the ground 35 seconds later, with what velocity was the object fired?
5. Identify each of the following as true or false.

    (a) A point in the domain of f where $f'(x)$ does not exist is a critical point.

    (b) Every continuous function on a closed interval will have an absolute minimum and an absolute maximum.

    (c) If $f'(c) = 0$, f will have either a local maximum or a local minimum at c.

    (d) An inflection point is an ordered pair.

    (e) If $f'(c) = 0$ and $f''(c) > 0$ then c is a local minimum.

    (f) If $f''(c) = 0$ in the second derivative test, we must use some other method to determine if c is a local max or min.

    (g) A continuous function on $[a, b]$ will always have a local max or min at its endpoints.