Worksheet # 21: Optimization

1. Suppose that f is a function on an open interval $I = (a, b)$ and c is in I. Suppose that f is continuous at c, $f'(x) > 0$ for $x > c$ and $f'(x) < 0$ for $x < c$. Is $f(c)$ an absolute minimum value for f on I? Justify your answer.

2. A farmer has 2400 feet of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river. What are the dimensions of the field that has the largest area?

3. A hockey team plays in an arena with a seating capacity of 15000 spectators. With the ticket price set at \$12, average attendance at a game has been 11000. A market survey indicates that for each dollar the ticket price is lowered, average attendance will increase by 1000. How should the owners of the team set the ticket price to maximize their revenue from ticket sales?

4. An oil company needs to run a pipeline to a nearby station. The station and oil company are on opposite sides of a river that is 1 km wide, and that runs exactly west-east and the station is 10 km east along the river from the the oil company. The cost of building pipe on land is \$200 per meter and the cost of building pipe in water is \$300 per meter. Set up an equation whose solution(s) are the critical points of the cost function for this problem.

   Find the least expensive way to construct the pipe.

5. A flexible tube of length 4 m is bent into an L-shape. Where should the bend be made to minimize the distance between the two ends?

6. A 10 meter length of rope is to be cut into two pieces to form a square and a circle. How should the rope be cut to maximize the enclosed area?

7. Find the point(s) on the hyperbola $y = \dfrac{16}{x}$ that is (are) closest to $(0, 0)$. Be sure to clearly state what function you choose to minimize or maximize and why.

8. Consider a can in the shape of a right circular cylinder. The top and bottom of the can is made of a material that costs 4 cents per square centimeter, and the side is made of a material that costs 3 cents per square centimeter. We want to find the dimensions of the can which has volume 72 π cubic centimeters, and whose cost is as small as possible.

   (a) Find a function $f(r)$ which gives the cost of the can in terms of radius r. Be sure to specify the domain.

   (b) Give the radius and height of the can with least cost.

   (c) Explain how you known you have found the can of least cost.

9. Find the point on the line $y = x$ closest to the point $(1, 0)$. Find the point on the line $y = x$ closest to the point $(r, 1 - r)$. What does the collection of points $(r, 1 - r)$ look like graphically?

10. A box is to have a square base, no top, and a volume of 10 cubic centimeters. What are the dimensions of the box with the smallest possible total surface area? Provide an exact answer; do not convert your answer to decimal form. Make a sketch and introduce all the notation you are using.

MathExcel Worksheet # 21 Supplemental Problems

11. Wire of length 12 m is divided into two pieces and each piece is bent into a square. How should this be done in order to minimize the sum of the areas of the two squares?

12. Is there a point on the curve $y = x^2$ that is closest to the point $(0, 3)$? If so find this point; otherwise explain why there is not.

13. A poster of area 6000 cm^2 has blank margins of width 10 cm on the top and bottom and 6 cm on the sides. Find the dimensions that maximize the printed area.

14. Two non-negative numbers have sum 60. Can you find two such numbers such that their product is as small as possible? If so, find the two numbers; otherwise explain why you cannot.