Worksheet # 16: Review for Exam II

1. (a) State the definition of the derivative of a function $f(x)$ at a point a.

   (b) Find a function f and a number a such that

   $$\frac{f(x) - f(a)}{x - a} = \frac{\ln(2x - 1)}{x - 1}$$

   (c) Evaluate the following limit by using (a) and (b),

   $$\lim_{x \to 1} \frac{\ln(2x - 1)}{x - 1}$$

2. State the following rules with the hypotheses and conclusion.

   (a) The product rule

   (b) The quotient rule.

   (c) The chain rule.

3. A particle is moving along a line so that at time t seconds, the particle is $s(t) = \frac{1}{3}t^3 - t^2 - 8t$ meters to the right of the origin.

   (a) Find the time interval(s) when the particle's velocity is negative.

   (b) Find the time(s) when the particle's velocity is zero.

   (c) Find the time interval(s) when the particle's acceleration is positive.

   (d) Find the time interval(s) when the particle is speeding up. Hint: What do we need to know about velocity and acceleration in order to know that the derivative of the speed is positive?

4. Compute the first derivative of each of the following functions:

   (a) $f(x) = \cos(4\pi x^3) + \sin(3x + 2)$

   (b) $b(x) = x^4 \cos(3x^2)$

   (c) $y(\theta) = e^{\sec(2\theta)}$

   (d) $k(x) = \ln(7x^2 + \sin(x) + 1)$

   (e) $u(x) = (\arcsin(2x))^2$

   (f) $h(x) = \dfrac{8x^2 - 7x + 3}{\cos(2x)}$

   (g) $m(x) = \sqrt{x} + \dfrac{1}{\sqrt[3]{x^4}}$

   (h) $q(x) = \dfrac{e^x}{1 + x^2}$

   (i) $n(x) = \cos(\tan(x))$

   (j) $w(x) = \arcsin(x) \cdot \arccos(x)$

5. Let $f(x) = \cos(2x)$. Find the fourth derivative of f at $x = 0$, $f^{(4)}(0)$.

6. Let f be a one to one, differentiable function such that $f(1) = 2$, $f(2) = 3$, $f'(1) = 4$ and $f'(2) = 5$. Find the derivative of the inverse function, $(f^{-1})'(2)$.

7. Suppose the tangent line to $f(x)$ at $x = 3$ is given by $y = 2x - 4$. Find the tangent line to $g(x) = \dfrac{x}{f(x)}$ at $x = 3$. Put your answer in slope-intercept form.

8. Consider the curve $xy^3 + 12x^2 + y^2 = 24$. Assume this equation can be used to define y as a function of x near the point $(1, 2)$. Find the equation of the tangent line to this curve at $(1, 2)$.

9. Each side of a square is increasing at a rate of 5 cm/s. At what rate is the area of the square increasing when the area is 14 cm^2?

10. The sides of a rectangle are varying in such a way that the area is constant. At a certain instant the length of a rectangle is 16 m, the width is 12 m and the width is increasing at 3 m/s. What is the rate of change of the length at this instant?

11. Suppose f and g are differentiable functions such that $f(2) = 3$, $f'(2) = -1$, $g(2) = \frac{1}{4}$, and $g'(2) = 2$. Find:

   (a) $h'(2)$ where $h(x) = \ln([f(x)]^2)$;

   (b) $l'(2)$ where $l(x) = f(x^3 \cdot g(x))$.

12. Abby is driving north along Ash Road. Boris driving west on Birch Road. At 11:57 am, Boris is 5 km east of Oakville and traveling west at a speed of 60 km/h and Abby is 10 km north of Oakville and traveling north at a speed of 50 km/h.

   (a) Make a sketch showing the location and direction of travel for Abby and Boris.

   (b) Find the rate of change of the distance between Abby and Boris at 11:57 AM.

   (c) At 11:57 AM, is the distance between Abby and Boris increasing, decreasing, or not changing?

Math Excel Supplemental Problems # 16: Review for Exam II

1. Compute the first derivative of $h(x) = (e^{2x - \frac{5}{3}x^3}) \cdot \ln(2x^3 - 7)$.

2. Calculate the derivative of $\sin(\ln(3x^2y^4)) = \frac{x}{y}$ with respect to x.

3. Find the derivative of $Ax^2 + By^2 = C$, with respect to y.

4. The growth rate of the population in a bacteria colony at time t obeys the *differential equation* $P'(t) = kP(t)$ where k is a constant and t is measured in years.

   (a) Let A be a constant. Show that the function $P(t) = Ae^{kt}$ satisfies the differential equation.

   (b) If the colony initially has 100 bacteria and two years later has 200 bacteria, determine the values of A and k.

   (c) Suppose $P(t) = 100e^{.001t}$. When will the colony have 100,000 bacteria?

5. Suppose $x = \tan(y)$. If $-\dfrac{\pi}{2} < y < \dfrac{\pi}{2}$, we may define the inverse function $y = \arctan(x)$. Use implicit differentiation to find the derivative of $\arctan(x)$. [Hint: use a trigonometric identity.]