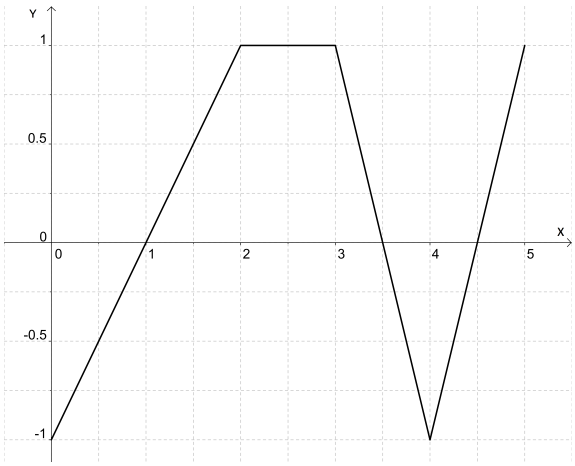
Worksheet # 10: The Derivative as a Function, Polynomials and Exponentials

Interesting Fact: Derivatives play a fundamental role in modeling physical problems. As one example, to mathematically describe elastic surfaces requires careful consideration of certain equations involving derivatives called "differential equations." One of the pioneers in elasticity theory was Sophie Germain, who in 1816 was the first woman to win a prize from the Paris Academy of Sciences for her pioneering paper "Recherches sur la théorie des surfaces élastiques." Germain is most famous for her work in number theory, and her award of the Paris Academy Prize inspired her to continue her work on what is now called "Fermat's Last Theorem". Germain's achievements are particularly impressive because her parents did not approve of women studying mathematics. At night, they would deny her warm clothing or a fire in her bedroom to prevent her from studying, but she hid candles and quilts and taught herself anyway.

1. Consider the graph below of the function $f(x)$ on the interval $[0, 5]$.



   (a) For which x values would the derivative $f'(x)$ not be defined?

   (b) Sketch the graph of the derivative function f'.

2. Water temperature affects the growth rate of brook trout. The table shows the amount of weight gained by brook trout after 24 days in various water temperatures.

Temperature (Celsius)	15.5	17.7	20.0	22.4	24.4
Weight gained (grams)	37.2	31.0	19.8	9.7	-9.8

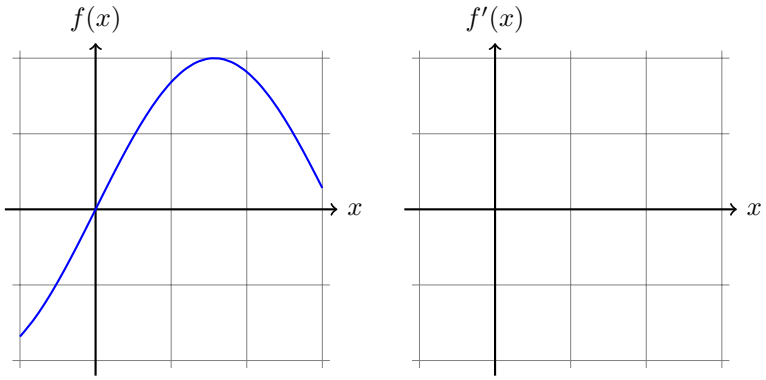
   (a) If $W(x)$ is the weight gain at temperature x, construct a table of estimated values for W'.

(b) Plot the points for $W(x)$ and $W'(x)$ in the $x-y$-plane. Sketch one possible version of the graph for W, and use this to sketch a possible version of the graph for W'.

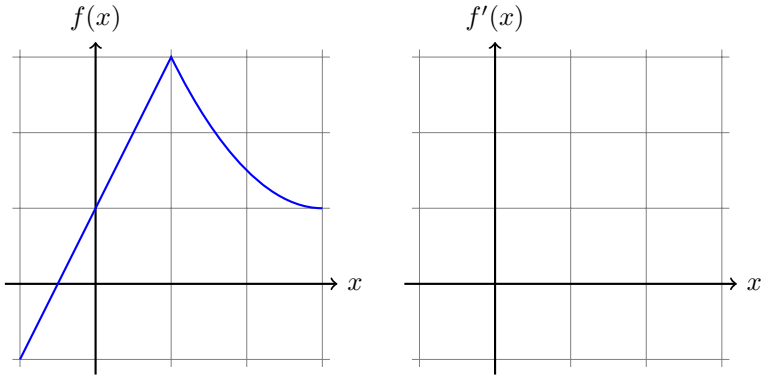
(c) What are the units for W'?

3. For each function f whose graph is given below, identify points where $f'(x)$ does not exist and sketch the graph of f'.

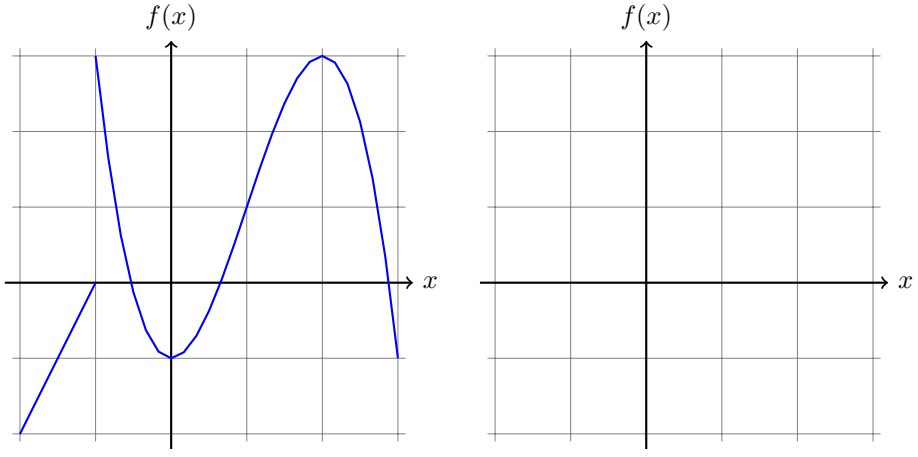
(a)



(b)



(c)



4. Compute the derivative of the following functions using both the limit definition and the rules for polynomials and exponentials.

(a) $f(x) = 4 + 8x - 10x^3$

(b) $g(x) = -7x^2 + x - 2$

5. Suppose N is the number of people in the United States who travel by car to another state for a vacation this year when the average price of gasoline is p dollars per gallon. Do you expect dN/dp to be positive or negative? Explain your answer. What about d^2N/dp^2?

6. Find a formula for the n-th derivative of x^n.

7. Find the first, second, and third derivatives of the following functions using the rules for polynomials and exponentials.

   (a) $f(x) = 3^{30}$

   (b) $g(t) = (t+1)(t+2)(t+3)$

   (c) $h(a) = \dfrac{\sqrt{a} + a}{a^3}$

   (d) $y = e^{x+2} + 1$

   (e) $F(x) = \dfrac{2}{x^3} + 3e^x - x^7$

Supplemental Worksheet # 10

8. The tangent line to the graph of $f(x)$ at the point $x = 4$ is given by $y = 5x - 3$. Find an equation for the tangent line to $g(x) = 2f(x) + 1$ at the point $x = 4$.

9. Find all values a such that the tangent line to $f(x) = \frac{1}{3}x^3 + x^2$ at the point a is parallel to the tangent line of $g(x) = x^2 + 9x + 100$.

10. Suppose that $f(x) = 2x + 7$.

    (a) Find $f'(x)$.

    (b) For which values of x is the tangent line to $f(x)$ parallel to $2x + y = 2$?

    (c) For each of the x-values found in part (b), find the equation of the tangent line to $f(x)$ at that value.