Worksheet # 1: Precalculus review: functions and inverse functions

Goals for MA 113 Recitations: Recitations are not homework help sessions! There are three goals for students in MA 113 recitations:

1. to develop your ability to make sense of problems and be persistent while solving them,

2. to develop your ability to productively collaborate with peers, and

3. to develop your ability to check your own work, i.e. to decide on your own whether or not your work is correct.

To help you reach these goals, you will spend the majority of the recitation working in small groups on worksheets. You are not expected to complete all of these problems — your TA will help guide you in selecting which problems to work on. Your focus should be to *discover your misunderstandings* by doing math collaboratively. Mistakes and misunderstandings are a normal part of learning mathematics — the only path to deep learning is to learn to effectively identify and revise our mistakes and misunderstandings.

Solutions to MA 113 worksheets are not provided. Instead, you should focus on using these problems to test your self-evaluation skills. Imagine you are taking an exam, and you need to check for yourself whether or not your work is correct — this is a skill you need to practice in order to do well! By collaborating with your peers and comparing solutions, with guidance and support from your TA, your problem solving and self-evaluation skills will improve. If there are worksheet problems that you are uncertain about, you are welcome to ask about them during recitation, during your TA or instructor office hours, at the Mathskeller, or at the Study.

1. Find the domain and range of $f(x) = \frac{x+1}{x^2+x-2}$.

2. For each of the following conditions, find the equation of the line that satisfies those conditions.

   (a) the line passes through the point $(1, 3)$ with slope 13.
   (b) the line passes through the points (π, π) and $(-8, -4)$.
   (c) the line has y-intercept 3 and has slope -2.

3. Let f be a linear function with slope m where $m \neq 0$. What is the slope of the inverse function f^{-1}? Why is your answer correct?

4. Consider the function whose graph appears below.
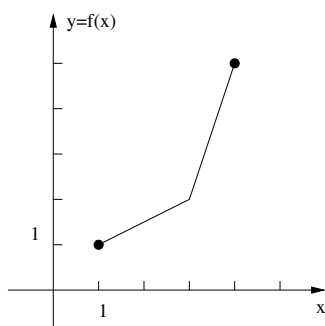
   (a) Find $f(3)$, $f^{-1}(2)$ and $f^{-1}(f(2))$.
   (b) Give the domain and range of f and of f^{-1}.
   (c) Sketch the graph of f^{-1}.

5. If $f(x) = 5x + 7$ and $g(x) = x^2$, find $f \circ g$ and $g \circ f$. Are the functions $f \circ g$ and $g \circ f$ the same function? Why or why not?

6. Let $f(x) = 2^{\cos(3(x+1)^2+9)} - 7$.

   (a) Can you find functions g and h such that $f = g \circ h$?

   (b) Can you find functions g, h, and s such that $f = g \circ h \circ s$?

   (c) Can you do this with four functions? Five functions? What is the largest number of functions you can find so that f can be written as a composition of those functions?

7. Let $f(x) = 2 + \frac{1}{x+3}$. Determine the inverse function of f, which we write as f^{-1}. Give the domain and range of f and the inverse function f^{-1}. Verify that $f \circ f^{-1}(x) = x$.

8. A ball is thrown in the air from ground level. The height of the ball in meters at time t seconds is given by the function $h(t) = -4.9t^2 + 30t$. At what time does the ball hit the ground? (Be sure to use the proper units!)

9. True or False: (justify your answer!)

   (a) Every function has an inverse.

   (b) If $f \circ g(x) = x$ for all x in the domain of g, then f is the inverse of g.

   (c) If $f \circ g(x) = x$ for all x in the domain of g and $g \circ f(x) = x$ for all x in the domain of f, then f is the inverse of g.

   (d) The function $f(x) = \sin(x)$ is one to one.

   (e) The function $f(x) = 1/(x+2)^3$ is one to one.

10. We form a box by removing squares of side length x centimeters from the four corners of a rectangle of width 100 cm and length 150 cm and then folding up the flaps between the squares that were removed. a) Write a function which gives the volume of the box as a function of x. b) Give the domain for this function.

11. Create a function that is the composition of ten functions. Can you do this in a "sneaky" way so that it is hard for someone else to figure out the ten functions you used? (Hint: try using different compositions of $g(x) = x + 1$ and $h(x) = 2x + 3$, for example $f = g \circ h \circ g \circ h \circ h$. What happens?)

Supplemental Worksheet # 1

1. Find the equation of the line that passes through $(1, 2)$ and is parallel to the line $4x + 2y = 11$. Put your answer in slope intercept form.

2. Write the equation of the line through $(2, 1)$ and $(-1, 3)$ in point slope form.

3. Write the equation of the line containing $(0, 1)$ and perpendicular to the line through $(0, 1)$ and $(2, 6)$.

4. The quadratic polynomial $f(x) = x^2 + bx + c$ has roots at -3 and 1. What are the values of b and c?

5. Let $f(x) = Ax^2 + Bx + C$. If $f(1) = 3$, $f(-1) = 7$, and $f(0) = 4$ what are the values of A, B and C?

6. Find the intersection of the lines $y = 5x + 10$ and $y = -8x - 3$. Remember that an intersection is a point in the plane, hence an ordered pair.

7. Recall the definition of the absolute value function:

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}.$$

   Sketch the graph of this function. Also, sketch the graphs of the functions $|x + 4|$ and $|x| + 4$.

8. True or False:

   (a) For any function f, $f(s + t) = f(s) + f(t)$.
   (b) If $f(s) = f(t)$, then $s = t$.
   (c) If $s = t$, then $f(s) = f(t)$.
   (d) A circle can be the graph of a function.
   (e) A function is a rule which assigns exactly one output $f(x)$ to every input x.
   (f) If $f(x)$ is increasing then $f(-52.55) \leq f(1752.0001)$.

9. Find A so that line $x + Ay = 1$ has slope 5. Can you find A so the line is horizontal? so that the line is vertical?

10. Let $f(x) = x^2 + 2x + 5$. Find the largest value of a so that f is one to one on the interval $(-\infty, a]$. Let g be the function f with the domain $(-\infty, a]$. Find the inverse function g^{-1}. Give the domain and range of g^{-1}.

11. Find the slope, x-intercept, and y-intercept of the line $3x - 2y = 4$.