MathExcel Worksheet N: Taylor Polynomials and Review

1. Use Taylor polynomials with $a = 0$ to approximate $\frac{1}{\sqrt[10]{e}}$ to five decimal places. Hint: Use a third degree Taylor polynomial.

2. (a) Use Taylor polynomials with $a = 0$ to approximate $\int_0^1 \sin(x^4)\,dx$ to five decimal places. Hint: Start with the fifth degree Taylor polynomial for $\sin(x)$ at $a = 0$.
   (b) Can you find an antiderivative for this integrand? Why or why not?

3. Use Taylor polynomials with $a = 0$ to approximate $\sin(1)$ to four decimal places.

4. (a) Use Taylor polynomials with $a = 0$ to approximate $\int_0^{0.5} x^2 e^{-x^2}\,dx$ to two decimal places. Hint: Use a fourth degree Taylor polynomial for $x^2 e^{-x^2}$.
   (b) Can you find an antiderivative for this integrand? Why or why not?

5. If $f(x) = (1 + x^5)^{1000}$, what are $f^{(273)}(0), f^{(999)}(0)$, and $f^{(824)}(0)$? Is $f^{(1000)}(0) = 0$?

Review Problems

6. Suppose $f(x) = x^2[g(x)]^3$, $g(4) = 2$, and $g'(4) = 3$. Find $f'(4)$.

7. Consider the function $h(x) = x^4 - 8x^2 + 16$ on the interval $[-4, 3]$. Find the absolute and local extrema of h on the interval $[-4, 3]$ and where they occur. Classify each point as a minimum or maximum.

8. Find the area of the region between the graphs of $y = x^2$ and $y = x^3$ for x in the interval $[0, 1]$. Hint: Can you express the area of this region in terms of the areas under the graphs of the two functions?

9. The velocity of a particle moving in a straight line is given by
$$v(t) = 3t^2 - 24t + 36$$
where t is in seconds and $v(t)$ is in meters per second.
   (a) Find the time intervals in $[0, 6]$ on which the particle is moving backwards and the intervals on which the particle is moving forwards.
   (b) Find the displacement of the particle over the time interval $[0, 6]$.
   (c) Find the total distance traveled by the particle over the time interval $[0, 6]$.

10. Find the number a such that the line $x = a$ bisects the area under the curve $y = 1/x^2$ with $1 \leq x \leq 4$.

11. Two cars start from an intersection at the same time. Car 1 travels north on a straight road at 20 mph and car 2 travels east on a straight road at 40 mph. Find the rate at which the distance between them is increasing after two hours.

12. Find $y(x)$ such that it satisfies the differential equation $y'(x) = 7y(x)$ with the initial condition $y(0) = 15$.

13. Consider the function
$$A(x) = \int_0^x \sec^2(t)\tan(t)\,dt.$$
Find the equation of the tangent line to the graph of $A(x)$ at $x = \pi/4$.

14. Find the pair of positive numbers (x, y) satisfying $4x+y = 9$ that maximizes the function $M(x, y) = x^2 y$.

15. If $f(x) = \tan(x)$ and $g(x) = 2x^2 + x$, then find the derivative of $(f \circ g)(x)$.

16. Sketch the graph of an increasing function $f(x)$ such that both $f'(x)$ and $A(x) = \int_0^x f(t)dt$ are decreasing.