MathExcel Supplemental Worksheet M: FTC, Indefinite Integrals, and Substitution

1. Fill in the blanks to complete the statement of the Fundamental Theorem of Calculus.

   (a) FTC 1:

   If f is ______________________ on the interval ___________, then for all x in ___________ the function g defined by $g(x) = \int_{\_\_}^{\_\_} f(t)\, dt$ is differentiable on the interval ___________, and satisfies the equation ___________ $= f(x)$.

   (b) FTC 2:

   If f is ______________________ on the interval $[a, b]$, then $\int_a^b f(x)dx =$ ________________, where F is ______________________, i.e., F and f satisfy the equation ___________ $= f(x)$.

2. If $f(x) = \int_0^{\sin(x)} \sqrt{1+t^2}\, dt$ and $g(y) = \int_3^y f(x)dx$, find $g''\left(\frac{\pi}{6}\right)$.

3. If $f(1) = 12$, f' is continuous, and $\int_1^4 f'(x)dx = 17$, what is the value of $f(4)$?

4. What is the difference between a definite and an indefinite integral? Give an example of each.

5. Evaluate the following

   (a) $\int (x^{1.3} + 7x^{2.5})dx$.

   (b) $\int (\sin(x) + \sinh(x))dx$.

   (Note that the hyperbolic sine function is defined as $\sinh(x) = \frac{e^x - e^{-x}}{2}$.)

   (c) $\int_0^{\pi/3} \frac{\sin\theta + \sin\theta\tan^2\theta}{\sec^2\theta} d\theta$.

6. A honeybee population starts with 100 bees and increases at a rate of $n'(t)$ bees per week. What does $100 + \int_0^{15} n'(t)dt$ represent?

7. A bacteria population is 4000 at time $t = 0$ and its rate of growth is $1000 \cdot 2^t$ bacteria per hour after t hours. What is the population after 1 hour?

8. A particle moves along the x-axis with velocity $v(t) = t^{-2}$. At time $t = 1$, the particle is at the origin. Show that the particle will never pass the point $x = 1$. *Hint: Find the position equation.*

9. State the substitution rule for both definite and indefinite integrals. Use complete sentences.

10. Evaluate:

    (a) $\int x\sqrt{x^2+7}\, dx$.

    (b) $\int x^2 e^{x^3} dx$.

    (c) $\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$.

    (d) $\int_1^2 \frac{4x^3}{x^4+2}\, dx$.

11. Recall that a function $f(x)$ is odd if $f(-x) = -f(x)$ for all x and it is even if $f(-x) = f(x)$. Consider the functions $P(x) = x^3 - x$ and $Q(x) = 3x^2 + x^4$.

   (a) Determine the parity of the functions $P(x)$ and $Q(x)$, i.e., determine whether each function is odd or even.

   (b) Sketch the graphs of $P(x)$ and $Q(x)$. How can you also tell the parity of each function from its graph?

   (c) Evaluate the following integrals:

      i. $\displaystyle\int_0^1 P(x)\,dx$

      ii. $\displaystyle\int_{-1}^1 P(x)\,dx$

      iii. $\displaystyle\int_0^2 Q(x)\,dx$

      iv. $\displaystyle\int_{-2}^2 Q(x)\,dx$

   (d) Do you notice anything interesting about your answers to ii and iv in (c)? What do the parities of $P(x)$ and $Q(x)$ have to do with these answers?

   (e) Suppose that $k(x)$ is an odd function and $\displaystyle\int_0^a k(x)\,dx = N$. Find $\displaystyle\int_{-a}^a k(x)\,dx$. Defend your answer using the substitution rule.

   (f) Suppose that $l(x)$ is an even function and $\displaystyle\int_0^b l(x)\,dx = M$. Find $\displaystyle\int_{-b}^b l(x)\,dx$. Defend your answer using the substitution rule.