MathExcel Worksheet K: Review for Exam III

1. Rewrite the following limits as definite integrals:

   (a) $\lim_{n\to\infty} \sum_{i=1}^{n} \left[(2+\frac{i}{n})e^{(2+\frac{i}{n})} \cdot \frac{1}{n} \right]$

   (b) $\lim_{n\to\infty} \sum_{i=1}^{n} \left[(2+\frac{3i}{n})e^{(2+\frac{3i}{n})} \cdot \frac{3}{n} \right]$

   (c) $\lim_{n\to\infty} \sum_{i=1}^{n} \left[\sqrt{5-(3+\frac{2i}{n})^2} \cdot \frac{2}{n} \right]$

2. Recall the following formulas:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \qquad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

   Recall that R_n denotes the n^{th} right-endpoint approximation of the area under the graph of $f(x)$ and that $R_n = \Delta x \sum_{i=1}^{n} f(a+i\Delta x)$, where $\Delta x = \frac{b-a}{n}$.

   (a) Find R_4 for the function $f(x) = x^2 + 2x$ on the interval $[1,3]$.

   (b) Find a formula for R_n for any n for the function $f(x) = x^2 + 2x$ on the interval $[1,3]$.

   (c) Find $\lim_{n\to\infty} R_n$.

3. Find the most general antiderivative of $4x^3 + 2x^{-1} + e^x - \cos x - \sin x + 2$.

4. Knowing $\int_0^2 f(x)dx = 3$, $\int_2^5 2f(x)dx = 4$, $\int_0^2 g(x)dx = 1$, $\int_0^5 \frac{g(x)}{6}dx = 18$, compute the following definite integrals.

   (a) $\int_0^5 f(x)\,dx$

   (b) $\int_0^5 g(x)\,dx$

   (c) $\int_5^0 f(x)\,dx$

   (d) $\int_0^2 [f(x)+g(x)]\,dx$

   (e) $\int_2^5 [2f(x)+3g(x)]\,dx$

   (f) $\int_2^0 7f(x)dx + \int_2^5 g(x)dx$

5. Let $f''(x) = 6x + 3 - \frac{2}{x^3}$, and $f'(1) = 1$ and $f(1) = 0$. Find $f'(x)$ and $f(x)$.

6. Use L'Hopital's Rule to find the limits below:

   (a) $\lim_{x\to\infty} \frac{2x}{e^x}$

   (b) $\lim_{x\to-\infty} 2x \cdot e^x$

   (c) $\lim_{x\to a} \frac{x^2 - a(3x-2a)}{x^2-a^2}$

   (d) $\lim_{x\to 1} \frac{\sqrt{x}-1}{\ln x}$

7. Find the point on the line $y = x+3$ closest to the point (3,0). Be sure to explain why the point you found gives the minimum distance.

8. Determine all numbers c which satisfy the conclusions of the Mean Value Theorem for $f(x) = x^3 + 2x^2 - x$ on the interval $[-1,2]$.

9. Given the function:
$$f(x) = -\frac{1}{3}x^3 + \frac{3}{2}x^2 - 2x + 5$$
   (a) Find the critical points of $f(x)$.
   (b) State the First and Second Derivative Tests. Find the interval(s) where the graph of $f(x)$ is increasing and the interval(s) where the graph of $f(x)$ is decreasing.
   (c) Decide which of the points you found in part (a) are local maxima or minima of $f(x)$.
   (d) Find the inflection points of $f(x)$.
   (e) For the function above, find the interval(s) where the graph of $f(x)$ is concave up and the interval(s) where the graph of $f(x)$ is concave down.

10. Show that the polynomial $f(x) = 4x^5 + x^3 + 7x - 24$ has at least one real root. *Hint*: Use the Intermediate Value Theorem.

11. Suppose that the amount of money in a bank account after t years is given by
$$A(t) = 2000 - 10t \cdot e^{(5 - \frac{t^2}{8})}$$
Determine the minimum and maximum amount of money in the account during the first 10 years that it is open.

12. If $f(2) = 30$ and $f'(x) \geq 4$ for $2 \leq x \leq 6$, how small can $f(6)$ be?

13. Suppose a sculptor can sell 15 statues at \$500 each, but for each additional statue she makes, the price goes down by \$15 (they are becoming less trendy). How many statues should she produce to maximize her revenue? What is her maximum revenue?

14. Consider the function $f(x) = x^2 + 3$. We are interested in the area A under the graph of $f(x)$ on the interval $[1, 5]$.
   (a) Divide the interval $[1, 5]$ into n subintervals of equal length and write an expression for R_n, the sum that represents the right-endpoint approximation of the area A.
   (b) Use the formula $\sum_{i=1}^{N} i^2 = \frac{N(N+1)(2N+1)}{6}$ to find a closed expression for R_n.
   (c) Take the appropriate limit of R_n to find an exact value for the area A.

15. Suppose that an object is fired downward, with an unknown velocity, from a plane flying at 10,700 m. If the object strikes the ground 35 seconds later, with what velocity was the object fired?

16. Identify each of the following as true or false.
   (a) A point in the domain of f where $f'(x)$ does not exist is a critical point.
   (b) Every continuous function on a closed interval will have an absolute minimum and an absolute maximum.
   (c) If $f'(c) = 0$, f will have either a local maximum or a local minimum at c.
   (d) An inflection point is an ordered pair.
   (e) If $f'(c) = 0$ and $f''(c) > 0$ then c is a local minimum.
   (f) If $f''(c) = 0$ in the second derivative test, we must use some other method to determine if c is a local max or min.
   (g) A continuous function on $[a, b]$ will always have a local max or min at its endpoints.