

MathExcel Worksheet J: Optimization and Antiderivatives

- A designer wants to introduce a new line of bookcases: He wants to make at least 100 bookcases, but not more than 2000 of them. He predicts the cost of producing x bookcases is $C(x)$. Assume that $C(x)$ is a differentiable function. Which of the following must he do to find the minimum average cost, $c(x) = \frac{C(x)}{x}$?
 - Find the points where $c'(x) = 0$ and evaluate $c(x)$ there.
 - Compute $c''(x)$ to check which of the critical points in (I) give the absolute minimum.
 - Check the values of $c(x)$ at the end points of the domain.
 - I only
 - I and II only
 - I and III only
 - I, II and III.
- You are given a continuous function f for which $f''(x) > 0$ everywhere except at $x = a$. Can f have an absolute maximum at $x = a$? Give an example or justify your answer.
- Which of the following statements is true?

If f is a continuous function on $[a, b]$, then

 - there must be numbers m and M such that $m \leq f(x) \leq M$ for $x \in [a, b]$.
 - there must be local extreme values, but there may or may not be an absolute maximum or absolute minimum value for the function.
 - any absolute maximum or minimum would be at either the endpoints of the interval or at places in the domain where $f'(x) = 0$
- For each of the following, decide whether the statement is true or false. Justify your choice.
 - If $f''(a) = 0$, then f has an inflection point at a .
 - If a racer is running back and forth along a straight path and finishes the race at the place where she began then there was at least one moment, other than the beginning and the end of the race, when she was "stopped (i.e., her speed was 0).
 - An antiderivative of a product of functions, fg , is an antiderivative of f times an antiderivative of g .
- Is there a point (or points) on the curve $y = x^2$ that is (are) closest to the point $(0, 3)$? If so, find the point(s). If not, explain why there is not.
- Let $f(x) = 1/x^2$, and $F(x)$ be an antiderivative of f with the property that $F(1) = 1$. Is $F(-3) = 3$? Why or why not?

7. If f is an antiderivative of g , and g is an antiderivative of h , then which of the following is true?
- (a) h is an antiderivative of f
 - (b) h is the second derivative of f
 - (c) h is the derivative of f''

8. Show that $F(x) = \frac{x^{n+1} - 1}{n + 1}$ is an antiderivative of $y = x^n$ for $n \neq -1$. Then use L'Hôpital's Rule to prove that

$$\lim_{n \rightarrow -1} F(x) = \ln(x)$$

In this limit, x is fixed and n is the variable. This result shows that, although the Power Rule breaks down for $n = -1$, the antiderivative $\ln(x)$ of $y = x^{-1}$ is a limit of antiderivatives of x^n as $n \rightarrow -1$.

9. Find an antiderivative for $f(x) = |x|$.
10. Compute the following indefinite integrals (Don't forget the constant C):

(a) $\int (5t^4 + 13t^2 - 7) dt$

(b) $\int \frac{1-x}{x} dx$

(c) $\int \sin(7x - 5) dx$

(d) $\int \sec(x + 5) \tan(x + 5) dx$

11. Suppose you are told that the acceleration function of an object is a continuous function $a(t)$. Lets say you are given that the velocity $v(0) = 1$. Can you find the position of the object at any time t ? Why or why not?
12. Consider the functions $f(x) = e^x$ and $g(x) = x^{1,000,000}$.
As $x \rightarrow \infty$ which of the following is true? Justify your answer using L'Hôpital's Rule.
- A. f grows faster than g .
 - B. g grows faster than f .
 - C. We cannot determine.
 - D. They grow at the same rate.
13. Water is being poured into a "Dixie cup" (a standard cup that is smaller at the bottom than at the top). The height of the water in the cup is a function of the volume of water in the cup. The graph of this function is
- (a) increasing and concave up
 - (b) increasing and concave down
 - (c) a straight line with positive slope.