# MathExcel Supplemental Problems H:
# More Applications of Derivatives

## 1 Extreme Value Theorem

1. State the Extreme Value Theorem.
2. Find all critical points of each function. Use these to find the absolute maximum, absolute minimum, and relative extrema on the given intervals.
   (a) $f(x) = (x-3)^2(x+4)^3, \qquad [0,4]$
   (b) $f(x) = (x-3)^2(x+4)^3, \qquad [-4,4]$
   (c) $g(x) = x^2 + 2x - \pi^2, \qquad [-1,2]$
   (d) $f(x) = \dfrac{1}{x^2 + 12x - 13}, \qquad [2,5]$
   (e) $g(x) = \dfrac{1}{x-1} - \dfrac{1}{x} \qquad [1/2, 2]$
   (f) $h(x) = x^5 - 80x \qquad [-3,3]$
3. Benny Snell runs a 4.56 40 yard dash. Is there necessarily a moment in time when Benny reaches his maximum velocity?
4. The function $f(x) = x^2$ on the open interval $(-2,1)$ has an absolute minimum at $x = 0$, but no absolute maximum. Why does this not contradict the Extreme Value Theorem?
5. Classify the absolute maxima and minima of the piecewise function
$$f(x) = \begin{cases} 0 & x = -2 \\ x^2 & -2 < x \leq 1 \end{cases}$$
   on the interval $[-2,1]$. How does your conclusion stack up against the EVT?
6. In algebra class, we learn that the vertex of a parabola $y = ax^2 + bx + c$ is given by $x = -b/2a$. Use calculus to show that this is true.
7. Consider a rectangle of length $x$ and width $y$ such that the perimeter is 20 units.
   (a) Write a function $A(x)$ for the area of the rectangle in terms of only $x$.
   (b) State the domain of $A(x)$, keeping in mind the physical limitations of the rectangle.
   (c) Find the critical points of $A(x)$.
   (d) Find the dimensions of such a rectangle with the maximum possible area.

## 2 Mean Value Theorem

8. State the Mean Value Theorem. When compared to Rolle's Theorem, what is different about the assumptions and the conclusion?
9. Jared's friend, Jeremy, has a need for speed. In an effort to arrive in Lexington to meet his buddy for a showing of the new *X-Men* movie, Jeremy must drive 83 miles from his workplace in Cincinnati, Ohio. He leaves work at 5pm and arrives at the theater at 6pm. Knowing that the speed limit on I-75 is 70mph for the entire trip, I claim that he should have received a ticket for speeding. Am I correct? Why or why not?
10. Find all values $c$ that satisfy the conclusion of the MVT for each function on the given interval.

(a) $f(x) = \sqrt{x}$ $\quad [9, 25]$

(b) $f(x) = x\ln(x)$ $\quad [1, 2]$

(c) $f(x) = x^3$ $\quad [-4, 5]$

11. Let $f(x) = (x-3)^{-2}$. Show that there is no value $c$ in the interval $(1, 4)$ such that

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}.$$

Why is this not a contradiction to the Mean Value Theorem?

12. Suppose $f(x)$ is a differentiable function on $(a, b)$ and $f'(x) = 0$ for each $x$ in $(a, b)$. Use the Mean Value Theorem to show that $f(x)$ must be a constant function on $(a, b)$.

13. Consider the function $f(x) = -\cos(\pi x)x^3$. Use the Mean Value Theorem to show that the equation

$$\pi\sin(\pi x)x^3 - 3\cos(\pi x)x^2 = 9$$

has a solution on the interval $(0, 3)$. Note that it is no easy task to solve this equation algebraically, but the MVT guarantees that it does have a solution in $(0, 3)$.

14. For the function $f(x) = |2x|$, show that there is no point $c$ on the interval $(-1, 1)$ where

$$f'(c) = \frac{f(1) - f(-1)}{1 - (-1)}.$$

Why does this not contradict the MVT?

## 3 Exponential Growth and Decay

15. Find a differential equation which has $f(x) = 10e^{6t}$ as a solution.

16. Find two different solutions to the differential equation $\dfrac{dy}{dt} = 14y$.

17. Mario fears that the goomba population is growing exponentially. His notes suggest that for every goomba that exists now, there will be 2 more next year.

    (a) Let $G(t)$ represent the number of goombas in the Mushroom Kingdom at year $t$. If there are 580 goombas in the kingdom at year $t = 0$, give an expression for $G(t)$.

    (b) Model this dire situation with a differential equation. Check that your answer to part (a) satisfies the differential equation.