Math Excel Worksheet G: Review for Exam II

1. True or False:

   (a) For a differentiable function $f(x)$ and a constant c, the derivative of $cf(x)$ is $cf'(x)$.

   (b) For two differentiable functions $f(x)$ and $g(x)$, the derivative of $f(x) \cdot g(x)$ is $f'(x) \cdot g'(x)$.

   (c) For three differentiable functions $f(x)$, $g(x)$, and $h(x)$, the derivative of $f(x) \cdot g(x) \cdot h(x)$ is $f'(x) \cdot g(x) \cdot h(x) + f(x) \cdot g'(x) \cdot h(x) + f(x) \cdot g(x) \cdot h'(x)$.

   (d) For two differentiable functions $f(x)$ and $g(x)$, the derivative of $(f \circ g)(x)$ is $f'(g(x)) \cdot g'(x)$.

   (e) For two differentiable functions $f(x)$ and $g(x)$, where $g(x) \neq 0$, the derivative of $\frac{f(x)}{g(x)}$ is $\frac{f'(x)}{g'(x)}$.

   (f) For two differentiable functions $f(x)$ and $g(x)$, where $g(x) \neq 0$, the derivative of $\frac{f(x)}{g(x)}$ is $\frac{f(x)g'(x)-f'(x)g(x)}{(g(x))^2}$.

   (g) The derivative of a polynomial is a polynomial.

2. A hot air balloon rising vertically is tracked by an observer located 5 km from the liftoff point.

   (a) Find an equation to relate the height of the balloon and the angle of the observer's line of sight.

   (b) At a certain moment, the angle between the observer's line of sight and the ground is $\frac{\pi}{3}$ rad, and the angle is changing at a rate of 0.1 rad/min. How fast is the balloon rising?

3. A 5 meter ladder leans against a wall. The bottom of the ladder is 3 meters away from the wall at $t = 0$ seconds and slides away from the wall at a rate of 0.3 meters per second. Find the velocity of the top of the ladder at time $t = 0$ seconds.

4. A car drives at 40 miles per hour on a straight road. There is a house 3 miles away from the road. What is the rate of change in the angle between the house and the car, and the house and the road when the car passes the house?

5. Let $f(x) = \frac{1}{(x-3)^2}$. Compute $f'(x)$ using the limit definition of derivative.

6. Find a function f and a number a so that the limit

$$\lim_{h \to 0} \frac{e^{(1+h)^2} - e}{h}$$

   represents a derivative $f'(a)$.

7. Find a function f and a number a so that the limit

$$\lim_{\theta \to \pi} \frac{\cos(\theta) - \sin(\theta) + 1}{\theta - \pi}$$

   represents a derivative $f'(a)$.

8. If $f(x)$ is invertible, $g(x) = f^{-1}(x)$, and we have $f(3) = 7$ and $f'(3) = \sqrt{2}$, find the slope of the tangent line to $g(x)$ at the point (7,3).

9. Consider the curve described by the equation $\sin(y^2 e^y) = x^2 + y - 1$. Find an equation of the line tangent to this curve at the point (-1,0).

10. Find $f(4)$ and $f'(4)$ if the tangent line to the graph of $f(x)$ at $x = 4$ has the equation $y = 3x - 14$.

11. Compute the derivative of each of the following functions:
    (a) $f(x) = (2\pi)^{x^2} + \frac{1}{\sqrt{x}}$
    (b) $f(x) = 8^{2x-\frac{3}{x}}$
    (c) $g(x) = ln(\frac{x^2-2x+1}{x}$ (Note: You may want to simplify using log rules before differentiating.)

12. Suppose that $f(x) = 3\cos(x) - \sin(2x)$. Find $f^{(7)}(x)$.

13. Compute the first two derivatives of each function:
    (a) $l(x) = \sec(7x)$
    (b) $m(x) = \cot(x^2)$

14. You are inflating a spherical balloon at a rate of $\frac{3}{2}\pi$ $\mathbf{cm}^3$/sec.
    (a) How quickly is the radius changing when $r = 7$cm?
    (b) Is the radius increasing or decreasing? Explain.
    (c) Using your result from (a), determine how quickly the surface area of the balloon is changing when $r = 7$cm.
    (d) Is the surface area increasing or decreasing? Explain.
    (e) Having filled the balloon with air to a radius of 12cm, you notice a small hole from which air is escaping at a rate of $\frac{\pi}{2}$ $\mathbf{cm}^3$/sec. Determine how quickly the radius of the balloon is chancing when the volume of the balloon is 36π $\mathbf{cm}^3$. Explain the significance of the sign of your answer.

15. The height of an object falling with constant acceleration is given by a quadratic polynomial $h(t) = at^2 + bt + c$. An object is dropped (with an initial velocity of 0m/s) from a building that is 64m tall. The building is not on earth, but rather on a planet where the acceleration due to gravity is -8m/s^2.
    (a) Find the function $h(t)$ that gives the height of the object t seconds after it is dropped until it hits the ground.
    (b) How long will it take for the object to hit the ground?
    (c) Find the velocity of the object when it hits the ground.

16. Find all points on the graph of $3x^2 + 4y^2 + 3xy = 24$ where the tangent line is horizontal.

17. Find all values of x where $f^{(3)}(x) = 0$ if $f(x) = 17x^2 + 6x + xe^{2x} - 9$.