Math Excel Worksheet F: Review of Derivatives

1. Calculate the following derivatives.

   (a) $\dfrac{d}{dx}\ \ln(\cos(2\pi x))$

   (b) $\dfrac{d}{du}\ e^{-u^2}$

   (c) $\dfrac{d}{dt}\left(\dfrac{t}{t^2+1}\right)$

2. Let $f(x)$, $g(x)$, and $h(x)$ be differentiable functions such that $f(2) = 4$, $f'(2) = 5$, $g(\sqrt{2}) = 3$, $g'(\sqrt{2}) = \frac{1}{2}$, and $h(2) = \sqrt{2}$. Define $F(x) = (f(x)g(h(x)))$ and assume that $F'(2) = 1$. Find $h'(2)$.

3. Let $f(x) = \dfrac{\sqrt{6x^2-2}}{g(x)}$. If $g(1) = 2$ and $f'(1) = 3$, then what is the value of $g'(1)$? Explicitly state any derivative rules that you use, and make sure that you meet all of the criteria for using that rule. *Note: You may assume that* $\sqrt{4} = 2$.

4. Find the following derivatives:

   (a) $\dfrac{d}{dx}\sin\left(\tan\sqrt{1+x^3}\right)$

   (b) $\dfrac{d}{dx}\arctan\left(\dfrac{1}{\sqrt{x^2+1}}\right)$

   (c) $\dfrac{d}{dx}\sqrt{1+\sqrt{2+\sqrt{3+\sqrt{x}}}}$

5. Use the product and chain rules to find the following derivatives *without expanding the polynomial.* (You may recall working out the product rule for more than two functions; it is okay to use that here.)

   (a) $\dfrac{d}{dx}\left((x+1)(x+2)(x+3)\right)$

   (b) $\dfrac{d}{dx}\left((x+1)(x+2)^2(x+3)^3\right)$

6. Let $a_1, a_2, \dots, a_n$ be constants and define $f(x) = (x-a_1)(x-a_2)^2\cdots(x-a_n)^n$.

   (a) Find $f'(a_2)$. (Hint: think about the last problem!)

   (b) Verify that $f'(a_2) = f'(a_j)$ for $j = 3, 4, \dots, n$.

7. (a) State the limit definition of the derivative of the function $f(x)$.

   (b) Use the definition of the derivative and derivative rules to find $\dfrac{d}{dx}\sec(x)$. The following facts may be useful: $\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$, $\lim\limits_{h\to 0}\dfrac{\sin(h)}{h} = 1$, $\lim\limits_{h\to 0}\dfrac{1-\cos(h)}{h} = 0$.

   (c) Is the following equation true?

$$\lim_{y\to x}\frac{f(y)-f(x)}{y-x} = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

8. Consider the function

$$y = \frac{(x^2+1)^4}{(2x+1)^3(3x-1)^5}$$

   (a) Take the natural logarithm of both sides of the above equation. Use log rules to simplify your answer.

   (b) Use implicit differentiation to solve for $\frac{dy}{dx}$. Write the derivative using only the variable x.

   (c) Find $\frac{dy}{dx}$ using the quotient rule. Do the two expressions for the derivative agree? Which was easier to compute?

   (d) Find $\frac{dy}{dx}$ using the product rule with chain rule. Does your answer agree with the results of parts (b) and (c)? Which method was the easiest to use?

9. Find the equation of the tangent line to the curve $x^2 + 4xy + y^2 = 13$ at the point $(2, 1)$.

10. At what point on the curve $y = [\ln(x+4)]^2$ is the tangent horizontal?

11. Suppose $f'(x)$ exists for all x in (a, b). Which (if any) of the following statements are true?

   (I) $f(x)$ is continuous on (a, b).

   (II) $f(x)$ is continuous at $x = a$.

   (III) $f(x)$ is defined for all x in (a, b).

   (IV) $f(x)$ is differentiable on (a, b).

12. $\lim\limits_{x \to 0} \frac{\sin(x)}{x} = 1$ means that

   (a) $\frac{0}{0} = 1$

   (b) the tangent line to the graph of $y = \sin(x)$ at $(0, 0)$ is the line $y = x$

   (c) you can cancel the x's

   (d) $\sin(x) = x$ for x near 0.

13. Your mother says "If you eat your dinner, you can have dessert." You know this means, "If you don't eat your dinner, you cannot have dessert." Your calculus teacher says, "If f is differentiable at a, f is continuous at a." You know this means:

   (a) If f is continuous at a, f is differentiable at a

   (b) If f is not continuous at a, f is not differentiable at a.

   (c) If f is not differentiable at a, f is not continuous at a.

   (d) Knowing that f is not continuous at a does not give us enough information to deduce anything about whether the derivative of f exists at a.