MathExcel Supplemental Worksheet A: Functions and Inverse Functions

1. Find the domain of $f(x) = \sqrt{\dfrac{3x}{x^2 + x - 12}}$

2. Find all values of c such that $h(x) = \dfrac{x+3}{x^2 + 2cx + 4}$ has domain $\mathbb{R}$.

3. Suppose the graph of $f(x) = x^3$ is shifted down 8 units and to the left 2 units. What is the equation of the new graph?

4. Write an equation of the line *parallel* to $3x - 5y = 1$ which passes through the origin.

5. Find the point(s) of intersection of the line $y = 2x+2$ and the parabola given by $p(x) = x^2+3x-4$. Sketch a graph to illustrate your answer.

6. Determine if the following functions are even, odd, both, or neither. Explain.
   (a) $f(x) = x^3$.
   (b) $g(x) = x^4$.
   (c) $h(x) = 5$.
   (d) $k(x) = 0$.

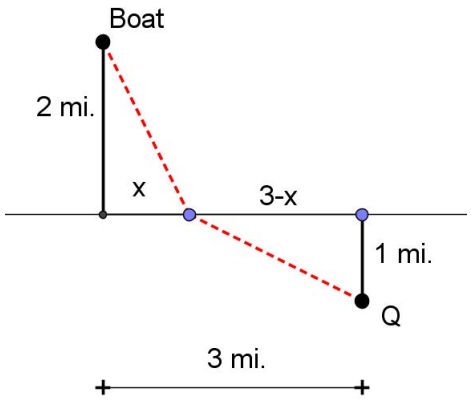
7. If $f(x)$ is odd and $f^{-1}(x)$ exists, show that $f^{-1}(x)$ is odd.

8. Use completing the square to find the roots of $f(x) = 4x^2 + x - 3$.

9. Show by completing the square that $f(x) = x^2 + bx + c$ is a translation of the function $R(x) = x^2$. By how many units has it been shifted horizontally? Vertically?

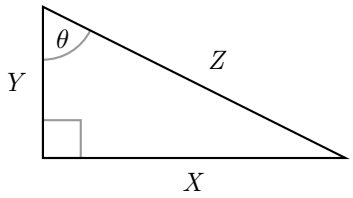
10. Let $f(x) = \sqrt{4x^2 - 4}$.
    (a) *Explain* why $f(x)$ is *not* a one-to-one function.
    (b) State a domain in which the given function *is* one-to-one.
    (c) In the domain from part (b), find $f^{-1}(x)$.

11. You are in a boat 2 miles from the nearest point on the coast. You are to go to a point Q located 3 miles down the coast and 1 mile inland (see figure). You can row at 2 miles per hour and walk at 4 miles per hour. Express the total time T of the trip as a function of x.



12. Use the Pythagorean theorem to determine the distance between points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$. Sketch a picture.

13. Consider the following triangle.



(a) Write down the Pythagorean Theorem as it pertains to this triangle.

(b) In terms of the variables in the image, what is $\sin(\theta)$? What is $\cos(\theta)$?

(c) In terms of the variables in the image, write down $\sin^2(\theta) + \cos^2(\theta)$ and simplify.

(d) Use part (c) to find an identity between $\tan(\theta)$ and $\sec(\theta)$.

(e) Use part (c) to find an identity between $\cot(\theta)$ and $\csc(\theta)$.

14. Find the domain and range of $f(\theta) = tan(\theta)$. Is this an invertible function? Why?

15. Given the function $y = \ln(ax^2 + 2x + 1)$.

(a) Find all real numbers a such that the domain of the function is $\mathbb{R}$.

(b) Find all real numbers a such that the range of the function is $\mathbb{R}$.

16. Which of the following equations are false? If the equation is false change it so that it becomes a true logarithmic identity.

(a) $\log_a(x) + \log_a(y) = \log_a(x + y)$

(b) $\frac{\log_a(x)}{\log_a(y)} = \log_a(x - y)$

(c) $b\log_a(x) + b\log_a(y) = \log_a((xy)^b)$

(d) $\log_a(x)\log_a(y) = \log_a(xy)$